A.2B Write Linear Equations in Various Forms

Definitions:

Linear Equation in Two Variables – a relationship with a constant rate of change
Represented by a graph that forms a straight line.

Various Forms of Linear Equations in Two Variables

Slope-Intercept Form	**Point-Slope Form**	**Standard Form**
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$
m is the slope b is the y-intercept	m is the slope (x_1, y_1) is a given point	Traditional format: A, B, C $\epsilon\ Z, A\ \geq 0$ x and y terms are on one side of the equation and the constant is on the other side.

Rate of Change by Various Methods –

Tabular	**Graphical**	**Algebraic**
m = $\frac{\text{change in y}-\text{values}}{\text{change in x}-\text{values}}$ m = $\frac{\Delta y}{\Delta x}$ m = $\frac{y_2 - y_1}{x_2 - x_1}$	slope = $\frac{\text{rise}}{\text{run}}$	$y = mx + b$ Solve equation for y. Slope is represented by m.

A line has a slope of $-\frac{2}{3}$ and passes through point (–9, 4). Write a representative equation of the line in slope-intercept form, point-slope form, and standard form.
Point-slope form, $y - y_1 = m(x - x_1)$ Using the point and the slope: $x_1 = -9$ $y_1 = 4$ $m = -\frac{2}{3}$ $y - y_1 = m(x - x_1)$ Equation in point-slope form $y - 4 = -\frac{2}{3}(x + 9)$
Transform the point-slope equation to determine the other representations.

Slope-intercept form, $y = mx + b$	Standard form, $Ax + By = C$
$y - 4 = -\frac{2}{3}(x + 9)$ $y - 4 = -\frac{2}{3}x - 6$ $y = -\frac{2}{3}x - 2$	$y - 4 = -\frac{2}{3}(x + 9)$ $y - 4 = -\frac{2}{3}x - 6$ $3(y - 4) = 3\left(-\frac{2}{3}x - 6\right)$ $3y - 12 = -2x - 18$ $2x + 3y = -6$

A line passes through points (–3, 2) and (5, 6). Write the representative equation of the line in slope-intercept form, point-slope form, and standard form.
Point-slope form, $y - y_1 = m(x - x_1)$ Use the two points to determine the slope. $m = \frac{6-2}{5-(-3)}$ $m = \frac{4}{8}$ or $m = \frac{1}{2}$ Using the point (5, 6) and the slope: $x_1 = 5$ $y_1 = 6$ $m = \frac{1}{2}$ $y - y_1 = m(x - x_1)$ $y - 6 = \frac{1}{2}(x - 5)$
Transform the point-slope equation to determine the other representations.

Slope-intercept form, $y = mx + b$	Standard form, $Ax + By = C$
$y - 6 = \frac{1}{2}(x - 5)$ $y - 6 = \frac{1}{2}x - \frac{5}{2}$ $y = \frac{1}{2}x + \frac{7}{2}$	$y - 6 = \frac{1}{2}(x - 5)$ $y - 6 = \frac{1}{2}x - \frac{5}{2}$ $2(y - 6) = 2\left(\frac{1}{2}x - \frac{5}{2}\right)$ $2y - 12 = x - 5$ $-7 = x - 2y$ $x - 2y = -7$

1 Find an equation in point-slope form, slope-intercept form, and standard form of an equation that has a slope of $-\frac{3}{5}$ and passes through the point (-5, 11).

Let us first set up our equation in the point-slope form of $y - y_1 = m(x - x_1)$. Since we are given the slope and a point we will just need to plug in the values into the formula.

Point slope form - $y - y_1 = m(x - x_1)$ ➡ $y - 11 = -\frac{3}{5}(x - (-5))$ ➡ $\mathbf{y - 11 = -\frac{3}{5}(x + 5)}$

Next, in order to put the equation in slope-intercept form we need to solve for the y-value of the equation by first distributing(multiplying) the slope to the values inside the parenthesis and then bringing the constant on the left side of the equation over to the other side of the equation.

Slope-Intercept Form – $y = mx + b$

Step 1: Distribute

$$y - 11 = -\frac{3}{5}(x + 5)$$

$$y - 11 = -\frac{3}{5}x - 3$$

Step 2: Solve for y

$$\begin{array}{rcl} y - 11 &=& -\frac{3}{5}x - 3 \\ +11 & & +11 \\ \hline \mathbf{y} &=& \mathbf{-\frac{3}{5}x + 8} \end{array}$$

Lastly, we need to take our equation and make it into the standard form format. We have a couple of different options that we can take but let us first multiply the entire equation by the denominator of the slope then take the coefficient and x-variable to the other side of the equation

Standard Form – $Ax + By = C$; where $A, B, C \in Z, A \geq 0$

Step 1: Multiply by denominator of slope

$$y = -\frac{3}{5}x + 8$$

$$5(y = -\frac{3}{5}x + 8)$$

$$5y = -3x + 40$$

Step 2: x-variable and coefficient brought to other side of equation

$$\begin{array}{rcl} 5y &=& -3x + 40 \\ +3x & & +3x \\ \hline \mathbf{3x + 5y} &=& \mathbf{40} \end{array}$$

Since A = 3, ≥ 0 our equation is in the traditional standard format form.

Therefore, point-slope form - $\mathbf{y - 11 = -\frac{3}{5}(x + 5)}$

slope-intercept form - $\mathbf{y = -\frac{3}{5}x + 8}$

standard form - $\mathbf{3x + 5y = 40}$

2 Find an equation in point-slope form, slope-intercept form, and standard form of an equation that passes through the points (-6, 2) and (6, 7).

To begin we must first find the slope of our equation by using the two points given and using the following formula of $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Let (-6, 2) represent the points (x_2, y_2) and (6, 7) represent the point (x_1, y_1). Then,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-7}{-6-6} = \frac{-5}{-12} = \frac{5}{12}$$

Now we can repeat what we did in #1. Choose any point that you want. Both answers will suffice.

Point slope form - $y - y_1 = m(x - x_1) \Rightarrow \mathbf{y - 7 = \frac{5}{12}(x - 6)}$ or

$y - y_1 = m(x - x_1) \Rightarrow y - 2 = \frac{5}{12}(x - (-6)) \Rightarrow \mathbf{y - 2 = \frac{5}{12}(x + 6)}$

Next, in order to put the equation in slope-intercept form we need to solve for the y-value of the equation by first distributing(multiplying) the slope to the values inside the parenthesis and then bringing the constant over to the other side of the equation. We can use any of the equations that we have found above.

Slope-Intercept Form – $y = mx + b$

Step 1: Distribute

$$y - 7 = \frac{5}{12}(x - 6)$$

$$y - 7 = \frac{5}{12}x - \frac{5}{2}$$

Step 2: Solve for y

$$\begin{array}{rcl} y - 7 & = & \frac{5}{12}x - \frac{5}{2} \\ +7 & & +7 \\ \hline \mathbf{y} & \mathbf{=} & \mathbf{\frac{5}{12}x + \frac{9}{2}} \end{array}$$

or

Step 1: Distribute

$$y - 2 = \frac{5}{12}(x + 6)$$

$$y - 2 = \frac{5}{12}x + \frac{5}{2}$$

Step 2: Solve for y

$$\begin{array}{rcl} Y - 2 & = & \frac{5}{12}x - \frac{5}{2} \\ +2 & & +2 \\ \hline \mathbf{y} & \mathbf{=} & \mathbf{\frac{5}{12}x + \frac{9}{2}} \end{array}$$

Lastly, we need to take our equation and make it into the standard form format. We have a couple of different options that we can take but let us first multiply the entire equation by the denominator of the slope then take the coefficient and x-variable to the other side of the equation.

Standard Form – $Ax + By = C$; where $A, B, C \epsilon Z, A \geq 0$

Step 1: multiply by denominator of slope

$$y = \frac{5}{12}x + \frac{9}{2}$$

$$12(y = \frac{5}{12}x + \frac{9}{2})$$

$$12y = 5x + 54$$

Step 2: x-term and coefficient brought to other side of equation

$$\begin{array}{r} 12y = 5x + 54 \\ -5x \qquad\quad -5x \\ \hline -5x + 12y = 54 \end{array}$$

Step 3: Since $A \leq 0$, multiply equation by -1

$$-1(-5x + 12y = 54)$$

$$\mathbf{5x - 12y = -64}$$

Therefore, point-slope form - $\mathbf{y - 7 = \frac{5}{12}(x - 6)}$

or $\mathbf{y - 2 = \frac{5}{12}(x + 6)}$

slope-intercept form - $\mathbf{y = \frac{5}{12}x + \frac{9}{2}}$

standard form - $\mathbf{5x - 12y = -64}$

3 What is an equation in slope intercept form of a line that passes through the point (0, -4) and has a slope of $-\frac{2}{3}$?

This problem is quite simple as we are asked to put what we are given into slope-intercept form and we are given the slope and y-intercept. The points (0, -4) is the y-intercept since the y-intercept exists where x = 0.

Therefore, slope-intercept form – y = mx + b ⇨ $y = -\frac{2}{3}x - 4$