A.2C Write Linear Equations Given a Table, Graph, or Verbal Description

Definitions:

Linear Equation in Two Variables – a relationship with a constant rate of change
Represented by a graph that forms a straight line.

Various Forms of Linear Equations in Two Variables

Slope-Intercept Form	**Point-Slope Form**	**Standard Form**
y = mx + b	$y - y_1 = m(x - x_1)$	Ax + By = C
m is the slope b is the y-intercept	m is the slope (x_1, y_1) is a given point	Traditional format: A, B, C $\epsilon\ Z, A \geq 0$ x and y terms are on one side of the equation and the constant is on the other side.

Rate of Change by Various Methods –

Tabular	**Graphical**	**Algebraic**
m = $\frac{\text{change in y}-\text{values}}{\text{change in x}-\text{values}}$ m = $\frac{\Delta y}{\Delta x}$ m = $\frac{y_2 - y_1}{x_2 - x_1}$	slope = $\frac{\text{rise}}{\text{run}}$	y = mx + b Solve equation for y. Slope is represented by m.

Write a representative equation of the line for the table of data in slope-intercept form, point-slope form, and standard form.

x	y
–5	1
0	3
5	5

Slope-intercept form, $y = mx + b$

Sample Response:
Select two points and determine the slope:
Using (–5, 1) and (5, 5)

$$m = \frac{5-1}{5-(-5)}$$

$$m = \frac{4}{10} \text{ or } m = \frac{2}{5}$$

Since (0, 3) is given, 3 is the y-intercept.
$b = 3$
$y = mx + b$
Using the slope and the y-intercept:

$$y = \frac{2}{5}x + 3$$

Point-slope form, $y - y_1 = m(x - x_1)$

Sample Response:
Select two points and determine the slope:
Using (–5, 1) and (5, 5)

$$m = \frac{5-1}{5-(-5)}$$

$$m = \frac{2}{5}$$

Using the point (–5, 1) and the slope:
$y - y_1 = m(x - x_1)$
$x_1 = -5$
$y_1 = 1$

$$y - 1 = \frac{2}{5}(x - (-5))$$

$$y - 1 = \frac{2}{5}(x + 5)$$

Standard form, $Ax + By = C$

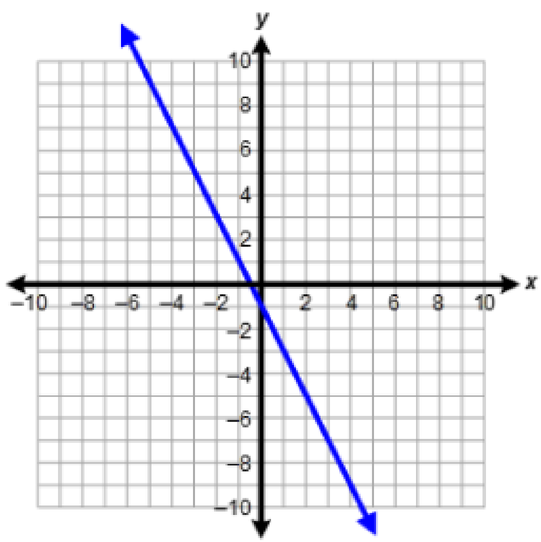
Sample Response:
Transform the slope-intercept equation:

$$y = \frac{2}{5}x + 3$$

$$5(y) = 5\left(\frac{2}{5}x + 3\right)$$

$5y = 2x + 15$
$2x - 5y = -15$

Write a representative linear equation for the line represented in the following graph in slope-intercept form, point-slope form, and standard form.



Slope-intercept form, $y = mx + b$

Sample Response:
Since the y-intercept (0, –1) is shown on the graph:
$b = -1$
Since the slope can be counted on the graph:

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

$m = -2$
Using the slope and y-intercept:
$y = -2x - 1$

Point-slope form, $y - y_1 = m(x - x_1)$

Sample Response:
Since the slope can be counted on the graph:

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

$m = -2$
Select any point on the graph:
Point (1, –3)
$y - y_1 = m(x - x_1)$
$y - (-3) = -2(x - 1)$
$y + 3 = -2(x - 1)$

Standard form, $Ax + By = C$

Sample Response:
Transform the slope-intercept equation:
$y = -2x - 1$
$2x + y = -1$

Kaylee is paid a base salary of \$50 per day plus \$2 for each crate she loads. If Kaylee loads 10 crates in a day, she makes \$70. Write a representative equation for the relationship between Kaylee's earnings per day, e, and the number of crates she loads, c. Represent the equation in slope-intercept form, point-slope form, and standard form.		
Slope-intercept form, $y = mx + b$	Point-slope form, $y - y_1 = m(x - x_1)$	Standard form, $Ax + By = C$
The base earnings represent the y-intercept; therefore, $b = 50$. The earnings per crates represents the slope; therefore, $m = 2$. $e = 2c + 50$	The earnings per crates represents the slope; therefore, $m = 2$. Since 10 crates a day makes \$70, the ordered pair is (10, 70). $m = 2$, point (10, 70) $y - y_1 = m(x - x_1)$ $e - 70 = 2(c - 10)$	Sample Response: Transform the slope-intercept equation: $e = 2c + 50$ $2c - e = -50$

Thomas is purchasing season tickets for his favorite baseball team. He is using a coupon that takes \$20 off of the purchase price of the tickets. Sales tax on the purchase is 5% of the price of the season tickets. Write a representative equation for the relationship between the final cost of the season tickets after discount and sales tax have been applied, C, and the price of the season tickets before discount and sales tax have been applied, t.
Sample Response: The cost of the season tickets after the discount is $(t - 20)$. The sales tax after the discount has been applied is $0.05(t - 20)$. The final cost of the season ticket purchase after discount and sales tax have been applied: $C = (t - 20) + 0.05(t - 20)$

Note: Please know that there are usually many different ways to solve an equation. If you watch the video lesson I will go through some of the different ways to solve the equations below including the use of technology.

1 The weight, *y*, in pounds, of a stack of books is dependent on the number of books, *x*, in the stack. This table below represents the weight of four different stacks of books.

Number of Books (x)	**Weight in Pounds (y)**
4	10
6	15
10	25
16	40

$2 = \Delta x$ $\Delta y = 5$

$6 = \Delta x$ $\Delta y = 15$

Let us first find the slope by using either $m = \frac{\Delta y}{\Delta x}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$ discussed below. We can see that from the first point to the second point located on the table that the change in y is 5 and the change in x is 2. Also, note from the third point to the fourth point the change in y is 15 and the change in x is 6. Therefore, we come to the same result regardless which points we choose. $m = \frac{\Delta y}{\Delta x} = \frac{15}{6} = \frac{5}{2}$

Write an equation point-slope, slope-intercept, and standard form in terms of *x* and *y* that represents the data in the table.

Using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ we can find our slope. You may choose any two points found in the table above to calculate the slope. Regardless, you will come up with the same result for the slope.

Let the point (6, 15) represent (x_2, y_2) and (4, 10) represent (x_1, y_1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-10}{6-4} = \frac{5}{2}$$

Now that we have found the slope we can now set up an equation in point-slope form. Use any of the points located in the table as any given point will work when writing an equation in point-slope form. For this problem I will write two equations in point-slope form and then use both examples to show that either equations will still give you the same result when rewriting the equation in slope-intercept form.

Example 1: Using the point (4, 10) and the slope $\frac{5}{2}$ to write in point-slope form.

$$y - y_1 = m(x - x_1) \Rightarrow \boldsymbol{y - 10 = \frac{5}{2}(x - 4)}$$

Example 2: Using the point (10, 25) and the slope $\frac{5}{2}$ to write in point-slope form.

$$y - y_1 = m(x - x_1) \Rightarrow \boldsymbol{y - 25 = \frac{5}{2}(x - 10)}$$

Now that we have our equations in point-slope form let us rewrite them in slope-intercept form of y = mx + b.

Example 1: $y - 10 = \frac{5}{2}(x - 4)$

Step 1: Distribute(multiply) the slope to the terms inside the parenthesis.

$$y - 10 = \frac{5}{2}(x - 4)$$

$$y - 10 = \frac{5}{2}x - 10$$

Step 2: Solve for y

$$y - 10 = \frac{5}{2}x - 10$$

$$\underline{\quad + 10 \qquad + 10}$$

$$\mathbf{y = \frac{5}{2}x}$$

Example 1: $y - 25 = \frac{5}{2}(x - 10)$

Step 1: Distribute(multiply) the slope to the terms inside the parenthesis.

$$y - 25 = \frac{5}{2}(x - 10)$$

$$y - 25 = \frac{5}{2}x - 25$$

Step 2: Solve for y

$$y - 25 = \frac{5}{2}x - 25$$

$$\underline{\quad + 25 \qquad + 25}$$

$$\mathbf{y = \frac{5}{2}x}$$

We can see that in either case when converting to slope-intercept form we will still come up with $\mathbf{y = \frac{5}{2}x}$

Lastly, we need to convert our equation from slope-intercept form to standard form.

Standard form: Ax + By =C

Step 1: Multiply all terms by the denominator of the slope.

$$y = \frac{5}{2}x$$

$$2(y = \frac{5}{2}x)$$

$$2y = 5x$$

Step 2: Move x and y terms to one side of the equation.

$$\begin{array}{r} 2y = 5x \\ \underline{-5x \qquad -5x} \\ -5x + 2y = 0 \end{array}$$

Step 3: Since A ≤ 0 we must distribute -1 to all terms in the equation.

$$-1(-5x + 2y = 0)$$

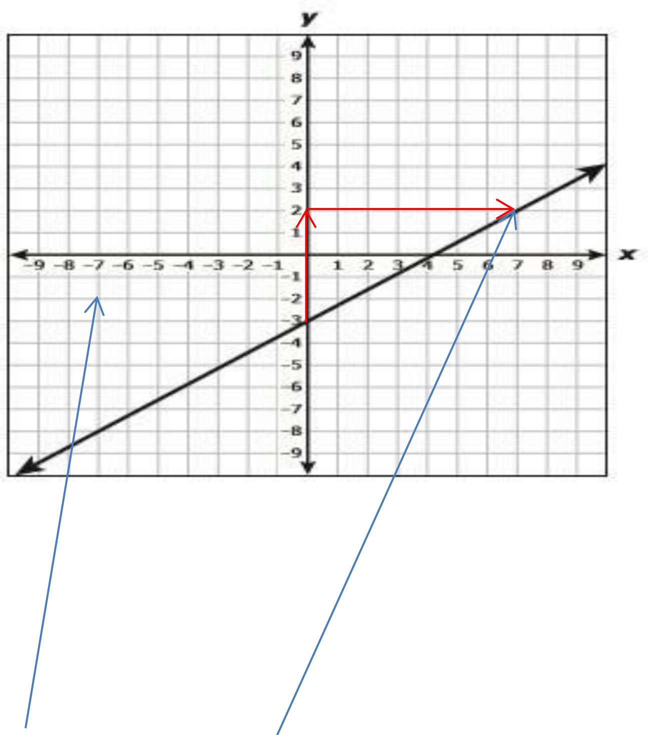
$$\mathbf{5x - 2y = 0}$$

Point-slope form - $\boldsymbol{y - 10 = \frac{5}{2}(x - 4)}$

Slope-intercept form - $\mathbf{y = \frac{5}{2}x}$

Standard form - **5x – 2y = 0**

2 The graph of a linear function is shown on the grid.



Let's use the method slope = $\frac{\text{rise}}{\text{run}}$ to find our slope and eliminate some of the answer choices. We need to first find two good points on the graph. Let us start from the point (0, -3) and end at the point (7, 2). As indicated by red arrows we can see that we rise up five units and go to the right 7 units.

slope $= \frac{\text{rise}}{\text{run}} = \frac{5}{7}$

Which equation is best represented by this graph?

A $y + 2 = \frac{7}{5}(x + 7)$

B $y - 2 = \frac{7}{5}(x - 7)$

C $y + 2 = \frac{5}{7}(x + 7)$

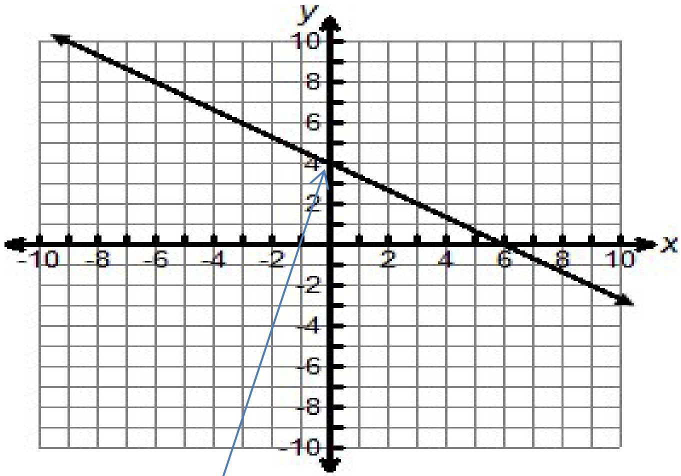
D $y - 2 = \frac{5}{7}(x - 7)$

Remember the form $y - y_1 = m(x - x_1)$ as all our answer choices are written in point-slope form. Since we found our slope to be $\frac{5}{7}$ we are able to eliminate answer choices A and B as they are representing a slope of $\frac{7}{5}$.

Investigating answer choice C we see the equation as $y + 2 = \frac{5}{7}(x + 7)$ which can also be written as $y - (-2) = \frac{5}{7}(x - (-7))$. This indicates that the point (-7, -2) should exist on the graph of our equation. However, this point is not on the graph of our equation. Therefore, we can eliminate answer choice C.

Lastly, answer choice D is represented by the equation $y - 2 = \frac{5}{7}(x - 7)$ which indicates that the point (7, 2) should exist on the graph of our equation. This point is located on the graph of our equation and the slope matches up. Therefore, answer choice D is the correct answer.

3 A linear function is graphed on the coordinate plane below.



Let's use the method slope = $\frac{\text{rise}}{\text{run}}$ to find our slope. We need to first find two good points on the graph. Let us start from the point (0, 4) and end at the point (3, 2). We can see that we rise down 2 units and go to the right 3 units.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{3} = -\frac{2}{3}$$

Which linear equation in two variables represents the same relationship?

F $y = -\frac{3}{2}x + 4$

G $y = -\frac{2}{3}x + 6$

H $2x + 3y = 12$

J $2x - 3y = 12$

We can easily find our y-intercept by looking at where the graph of our equation crosses the y-axis or noticing that our graph contains the point (0, 4). Therefore our y-intercept = 4. Using our slope of $-\frac{2}{3}$ and the y-intercept we can now find the equation of our graph in slope-intercept form, where m represents the slope and b represents the y-intercept.

$$y = mx + b$$
$$y = -\frac{2}{3}x + 4$$

Comparing what we found to answer choices F and G we can see that our equation does not match up. We will now need to convert our equation from slope-intercept form to standard form.

Step 1: Multiply all terms by the denominator of the slope.

$$y = -\frac{2}{3}x + 4$$

$$3(y = -\frac{2}{3}x + 4)$$

$$3y = -2x + 12$$

Step 2: Move x and y terms to one side of the equation.

$$\begin{array}{rl} & 3y = -2x + 12 \\ +2x & \underline{\qquad\quad 2x \qquad} \\ & \mathbf{2x + 3y = 12} \end{array}$$

Answer choice H is the correct answer.

4 Xavier plays an online video game that allows him to control many different characters. Each character starts the game with 200 gold coins and receives 30 gold coins for each month that passes within the game. If Xavier controls 50 characters and the number of months passed within the game is represented by x, which function can be used to determine the total number of gold coins that Xavier has collected in the game?

F $A(x) = 50(30x) + 200$

G $A(x) = 30x + 50(200)$

H $A(x) = 50(30x + 200)$

J $A(x) = 50 + (30x + 200)$

The number of months is represented by the variable x and the total amount of gold coins is represented by A(x). Since each character starts with 200 points at 0 months then our y-intercept is represented by the value 200. Each character then receives 30 points for each month that passes. Therefore, the value 30 is our rate of change or our slope. For one character the following equation can then be written in slope-intercept form.

$$A(x) = 30x + 200$$

However, Xavier has 50 characters and since each of his characters earns the same amount of gold coins we can then take our equation for one character and multiply it by 50 to include all 50 of his characters.

$$A(x) = 50(30x + 200)$$

Answer choice H will be the correct answer.