8.5E Solve Problems Involving Direct Variation

Definitions:

Direct Variation – a linear relationship between two variables, x (independent) and y (dependent), that always has a constant unchanged ratio, k, and can be represented by y = kx.

- Same as a linear proportional relationship
- Passes through the origin (0,0)
- Constant of proportionality represented as k = $\frac{y}{x}$
- k is also known as constant of variation
- Direct variation can also be phrased as direct proportion or directly proportional
- When b = 0 in y = mx + b, then k = the slope, m.

Rachel's parents live in Springfield, Colorado, which is approximately 780 miles from Rachel's home. If Rachel averages 60 miles per hour driving, how long will it take her to get from her home to her parents' home?

Solve the situation using a table, graph, and algebraic methods.

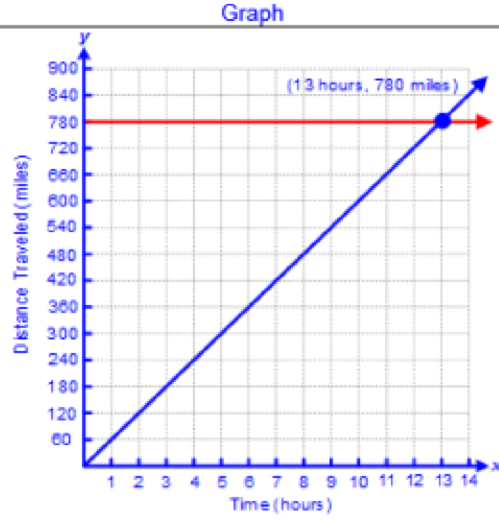
Sample solution

Table

Time (hrs)	Distance (mi)
1	60
6	360
7	420
8	480
9	540
10	600
11	660
12	720
13	780

It takes Rachel 13 hours to travel 780 miles.

Graph



Rachel travels 780 miles in 13 hours.

Algebraic

The equation $60x = 780$ can be used to represent this problem.

$$60x = 780$$

$$\frac{60x}{60} = \frac{780}{60}$$

$$x = 13$$

1 The value of y varies directly with x. When y= .45, x= .2

A. What is the value of x when y is 37.125?

B. What is the value of y when x is 8?

In the problem above we are given the point (.2, .45). Since we are told that y varies directly with x we know that the problem is direct variation. Therefore, our equation will be in the format of y = kx. Let us first find our k(constant of proportionality or also known as constant of variation). Since this is a direct variation problem in order to find our k we just need to take any given point and divide $\frac{y}{x}$.

$$k = \frac{y}{x} = \frac{.45}{.2} = 2.25$$

Therefore our equation y = kx ➡ y = 2.25x

To answer Part A of our equation let us plug in the given y-value of 37.125 and solve for our x-value.

Step 1: Plug-in known y-value

$$y = 2.25x$$

$$37.125 = 2.25x$$

Step 2: Solve for x

$$\frac{37.125}{2.25} = \frac{2.25x}{2.25}$$

16.5 = x

or **x = 16.5**

To answer Part B of our equation let us plug in the given x-value of 8 and solve for our y-value.

Step 1: Plug-in known x-value

$$y = 2.25x$$

$$y = 2.25(8)$$

Step 2: Solve for y

y = 18

A. x is 16.5 when y is 37.125

B. y is 18 when x is 8.

2 Gary's salary varies directly as the number of days he works. If his salary for five days is $42.00, how much would it be for 12 days?

We are given the point (12, 42).

This problem is very similar to the one we did above. Since salary varies directly with number of days Gary works then salary would be represented by the y-value and days would be represented by the x-value in our direct variation equation of y = kx. First let's solve for the k(constant of proportionality).

$$k = \frac{y}{x} = \frac{42}{12} = 3.5$$

Therefore our equation y = kx $\Rightarrow$ y = 3.5x

To finish solving our problem we are asked what his salary would be when Gary has worked for 12 days. Since x represents the days and y represents the salary in our equation we will just need to plug in the value 12 into the x of the equation and solve to find the salary.

y = 3.5(12) = 42

We have found that in 12 days Gary's salary would be **$42**.

3 In Physical Science class, Ernie discovered that a 4 pound weight stretched a coiled spring 14.2 inches and an 8 pound weight stretched the same spring 28.4 inches. Based on these two observations, he determined that the spring's amount of stretch varies directly with the amount of weight applied. What is the weight, in pounds, that will stretch the same coil 106.5 inches?

We are given the points (4, 14.2) and (8, 28.4).

We are told that the spring's amount of stretch varies directly with the amount of weight applied. Therefore, the spring's stretch would be represented by our y-value and weight would be represented by the x-value in our direct variation equation y = kx. Once again let us first solve for k(our constant of proportionality). We can use any of the two points that we are given to solve for k.

$k = \frac{y}{x} = \frac{14.2}{4} = 3.55$

or $k = \frac{y}{x} = \frac{28.4}{8} = 3.55$

Therefore our equation y = kx $\Rightarrow$ y = 3.55x

This time we are asked to find the weight in pounds the x-value if the coil is stretched 106.5 inches our y-value. This time we need to plug in our y-value of 106.5 into our equation and then solve for x.

Step 1: Plug-in known y-value

$$y = 3.55x$$

$$106.5 = 3.55x$$

Step 2: Solve for x

$$\frac{106.5 = 3.55x}{3.55 \qquad 3.55}$$

$$\mathbf{30 = x}$$

or $\mathbf{x = 30}$

Therefore, a **30** pound weight is being used when the coil is stretched 106.5 inches.