8.5B Represent Linear Non-Proportional Relationships

Definitions:

Linear Function – a relationship with a constant rate of change represented by a graph that forms a straight line in which each element of the input(x) is paired with exactly one element of the output(y).

Slope – the steepness of a line; rate of change in y(vertical) compared to change in x(horizontal).

Rate of Change by Various Methods – Slope

Tabular	Graphical	Algebraic
$m = \frac{\text{change in y–values}}{\text{change in x–values}}$ $m = \frac{\Delta y}{\Delta x}$ $m = \frac{y_2 - y_1}{x_2 - x_1}$	$\text{slope} = \frac{\text{rise}}{\text{run}}$	$y = mx + b$ Solve equation for y. Slope is represented by m.

Y-intercept – y coordinate of a point at which the relationship crosses the y-axis meaning the x coordinate is equal to zero, denoted as b in y = mx + b and the ordered pair (0, b).

Linear non-proportional relationship –

- Linear
- Does not pass through the origin (0, 0)
- Represented by y = mx + b, where $b \neq 0$

Verbal

There is a stack of cups on Sam's table. He measured one cup and found that the height of the cup from the bottom of the cup to beginning of the lip of the cup is 10.5 cm. The height of the lip of the cup is 2.2 cm.

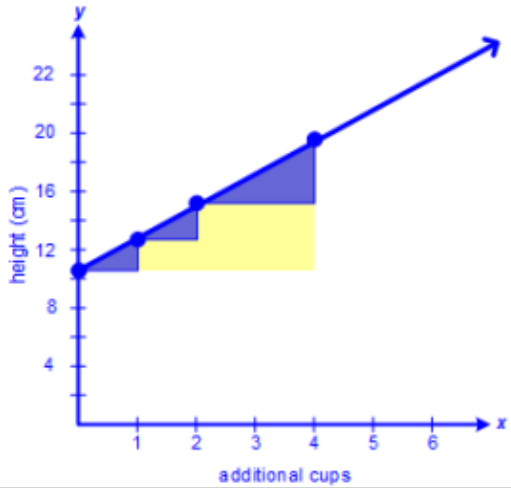
Table

The height of the stack depends on the number of additional cups; therefore, x is the number of additional cups and y is the height of the stack.

Additional Cups	Process	Height (cm)
0	0(2.2) + 10.5	10.5
1	1(2.2) + 10.5	12.7
2	2(2.2) + 10.5	14.9
4	4(2.2) + 10.5	19.3
x	x(2.2) + 10.5	y

The y-intercept of this linear non-proportional problem situation is 10.5 centimeters. The slope of this linear non-proportional problem situation is 2.2 centimeters for each additional cup.

Graphical



$$\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{12.7 - 10.5}{1 - 0} = \frac{2.2}{1} = 2.2$$

$$\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{14.9 - 12.7}{2 - 1} = \frac{2.2}{1} = 2.2$$

$$\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{19.3 - 14.9}{4 - 2} = \frac{4.4}{2} = \frac{2.2}{1} = 2.2$$

Algebraic

$y = 2.2x + 10.5$, where x represents the number of additional cups and y represents the height in centimeters.

Note: Please know that there are usually many different ways to solve an equation. If you watch the video lesson I will go through some of the different ways to solve the equations below including the use of technology.

1 The table below indicates a linear relationship between x and y.

x	y
2	5
4	9
6	13
12	25

Which equation would generate the values in this table?

A $y = x + 3$

B $y = 3x - 1$

C $y = 2x + 1$

D $y = 7 - x$

The first step into solving this equation algebraically would be to find the slope of the table above. We can do this by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where m represents the slope.

Pick any two points from the table as you will receive the same result pending you plug in and solve correctly. For this I will do three different examples to illustrate that you will receive the same slope.

Example 1: Let the point (2, 5) represent (x_2, y_2) and (4, 9) represent (x_1, y_1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 9}{2 - 4} = \frac{-4}{-2} = 2$$

Example 2: Let the point (6, 13) represent (x_2, y_2) and (12, 25) represent (x_1, y_1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 25}{6 - 12} = \frac{-12}{-6} = 2$$

Example 3: Let the point (12, 25) represent (x_2, y_2) and (2, 5) represent (x_1, y_1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 5}{12 - 2} = \frac{20}{10} = 2$$

Now that we have the slope of the equation we can plug the value into the formula $y = mx + b$. Therefore, now we have $y = 2x + b$. We still need to solve for the y-intercept of the equation. Once, again we can choose any point contained in the table and plug the values into the x and y values of the equation. Let us choose the point (2, 5).

First Step: Plug-in point into equation $y = 2x + b \Rightarrow 5 = 2(2) + b$

Second Step: Multiply $5 = 4 + b$

Third Step: Solve for b

$5 = 4 + b$

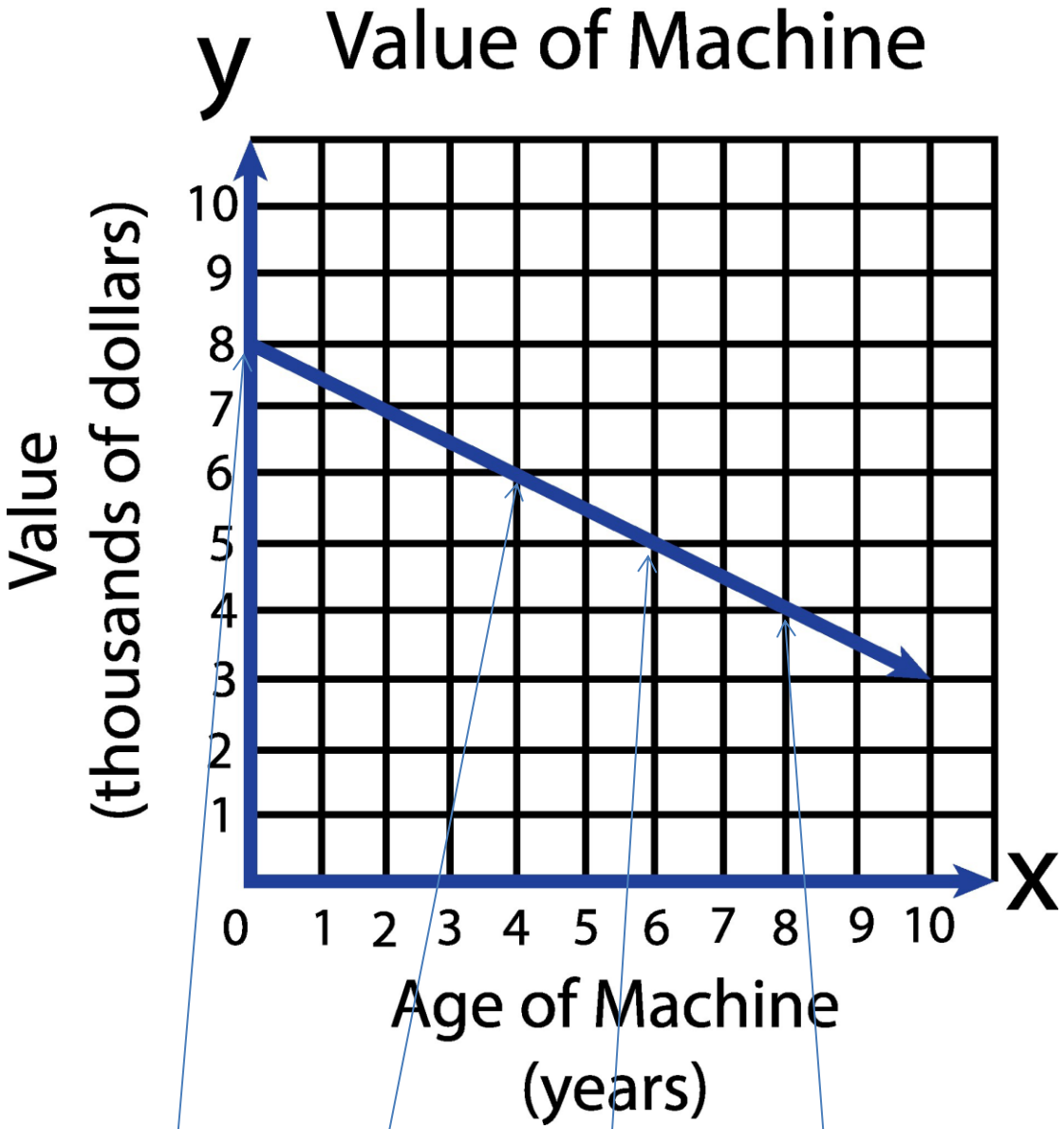
$-4 \quad -4$

$1 = b$

Last Step: Plug-in the value found for b into Equation. $y = 2x + 1$

Therefore the correct answer choice would be C.

2 The graph models the value of a machine over a 10-year period.



What is the equation in slope-intercept form that represents the relationship between x, the age of the machine in years, and y, the value of the machine in dollars over this 10-year period?

(0, 8000) (4, 6000) (6, 5000) (8, 4000)

In this problem let us first find the slope of the graph by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

First we must find two good points on the graph. When finding good points you want to preferable locate points where the graph crosses where the x and y grid lines intersect. It is not necessary but makes things easier as you will locate points involving integers and avoid fractions and/or decimals which may also be points where you are using best guess on the x and y values. You will only need two points and any two will do when trying to find the slope on a linear function as you will get the same result.

Let the point (6, 5000) represent (x_2, y_2) and (4, 6000) represent (x_1, y_1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5000 - 6000}{6 - 4} = \frac{-1000}{2} = -500$$

Now we just need to plug-in our values into the equation y = mx + b. Notice we were already given the y-intercept on the graph. Remember the y-intercept is the y-value where x = 0. Therefore, the point (0, 8000) let's us know that 8,000 is our y-intercept.

So, y = mx + b ⇒ **y = -500x + 8000** would be our equation for the graph above.

3 Jessica wants to go on a school trip next month. She already has $10 and plans to babysit for $5 an hour to earn the rest of the money.

Write an equation that could be used to determine *h*, the number of hours Jessica babysat and t, the total amount of money Jessica has earned.

We want to make the above verbal description into an equation in the form of y = mx + b. The slope(m) of our equation is the same as the rate of change of the verbal description. We are given the slope when told that she will earn $5 an hour. We are also given the y-intercept(a.k.a. our starting point) of $10 as this is the amount she currently has and the total amount Jessica has when she has worked 0 hours.

Our x-value, or in this case h, represents our independent variable as h represents the amount of hours Jessica has worked and the amount of money Jessica earns depends on the amount of hours she has worked. Therefore, our y-value, in this case t, represents our dependent variable.

So our equation in the form y = mx + b or ⇒ t = mh + b ⇒ **t =$5h + $10**

4 Which table contains only *v*-values and *w*-values where the value of *w* is 5 less than the product of *v* and 0.5?

Let us first translate the above verbal description into an equation form. So,

$$w = 0.5v - 5$$

Once again, there are a couple of different ways we can go about solving this problem but since all the tables have v-values of 6, 9, 12, and 17 let us create our own table with those values and then evaluate which table looks similar to ours.

F

v	*w*
6	8
9	9.5
12	11
17	13.5

G

v	*w*
6	3
9	4.5
12	6
17	8.5

H

v	*w*
6	−2
9	−0.5
12	1
17	3.5

J

v	*w*
6	0.5
9	2
12	3.5
17	6

v	**w = 0.5v – 5**	**w**
6	w = 0.5(6) – 5	-2
9	w = 0.5(9) – 5	-0.5
12	w = 0.5(12) – 5	1
17	w = 0.5(17) – 5	3.5

Therefore, when comparing the tables with v and w-values with our own table we see that the answer choice H will suffice.

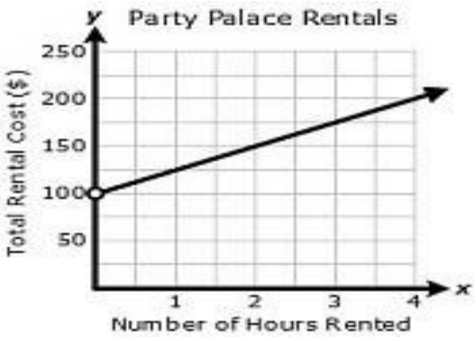
5 Party Palace offers inflatable rentals for a non-refundable deposit of $100 plus a rental fee of $25 per hour.

Which representation best shows the relationship between the total rental cost, *y*, and the number of hours the inflatable is rented, *x*?

Let us first take the verbal description given to us and represent it with a linear equation written in the form y = mx + b. We are letting y represent the total rental cost and x represent the number of hours the inflatable is rented. Since the rental fee is $25 an hour that will represent our rate of change or slope. The non-refundable deposit of $100 will represent our y-intercept(b) as this is the total amount of money we will need to initially pay or pay even when we have rented the inflatable for 0 hours. Therefore, our equation would be represented by

y = $25x + $100

A



Answer choice A shows that we start at an initial value of $100 and then at 1hour shows our total at $125, then at 2 hours shows $150. We can see that according to this graph our slope would be $25 and our y-intercept would be $100 which could be represented by the equation **y = $25x + $100**.

B

Party Palace Rentals

Hours Rented, *x*	Total Rental Cost, *y*
2	$225
4	$425
6	$625

If we use the equation we found above and evaluate the amount at 2 hours we will find that y = $25(2) + $100 = $150. This amount does not match the table to the right. This answer choice is **not** the correct option.

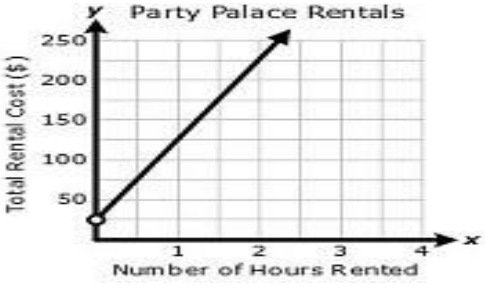
C

Party Palace Rentals

Hours Rented, *x*	Total Rental Cost, *y*
2	$50
4	$100
6	$150

If we use the same reasoning as we did for answer choice B we will see that this answer choice is **not** the correct option.

D



Answer choice B shows that we start at an initial value of $25 which would give us a y-intercept of $25 which does match our initial equation. This answer choice is **not** the correct option.