A.2A Domain and Range of a Linear Function

Definitions:

Linear function - a relationship with a constant rate of change represented by a graph that forms a straight line in which each element of the input(x) is paired with exactly one element of the output(y).

Domain – set of input values for the independent variable over which the function is defined. aka: all the "x-values".

Range – set of output values for the dependent variable over which the function is defined. aka: all of the "y-values".

Continuous function – function whose values are continuous or unbroken over the specified domain.

Discrete function – function whose values are distinct and separate and not connect; values are not continuous. Discrete functions are defined by their domain.

Inequality representations –

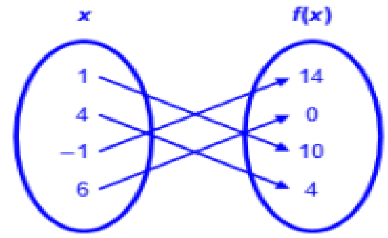
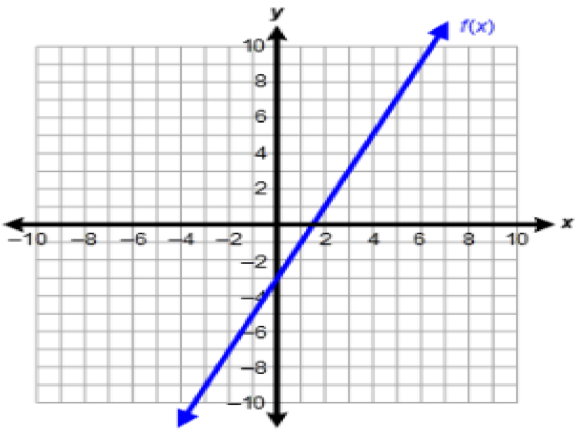
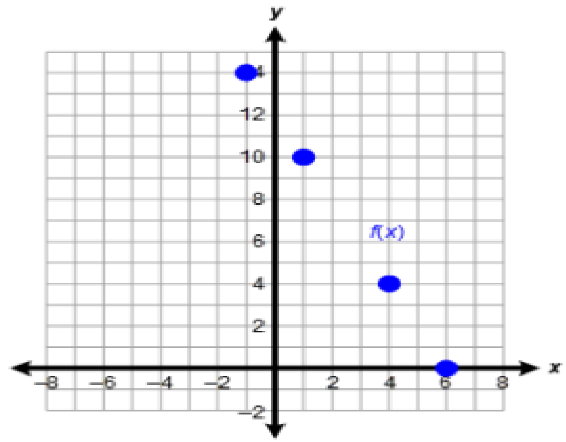
Verbal description	Inequality Notation
x is all real numbers less than five	$x < 5, x \in \Re$
x is all real numbers	x $\in \Re$
y is all real numbers greater than -3 and less than or equal to 6	$-3 < y \leq 6, y \in \Re$
y is all integers greater than or equal to 0	$y \geq 0, y \in \mathbb{Z}$

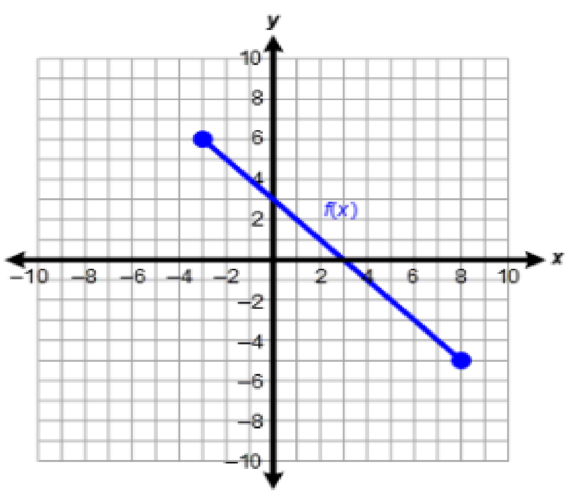
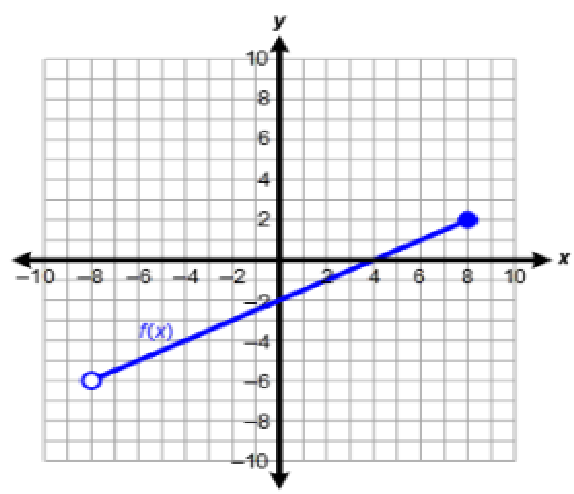
Note: Natural numbers are denoted by the symbol $\mathbb{N}$.

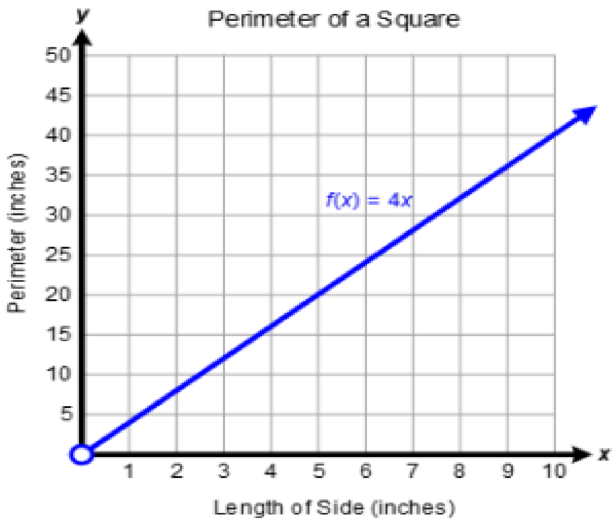
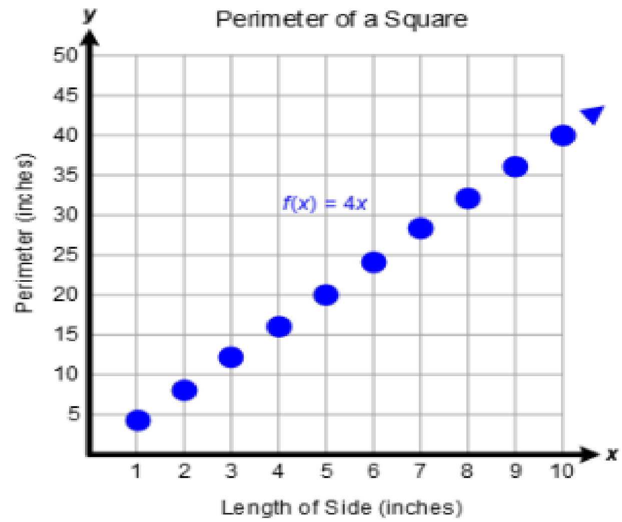
Whole numbers are denoted by the symbol W.

Integers are denoted by the symbol $\mathbb{Z}$.

Real numbers are denoted by the symbol $\Re$.

Continuous function	Discrete function
$f(x) = 2x - 3$	
	
Domain: All real numbers $x \in \Re$	Domain: $\{-1, 1, 4, 6\}$
Range: All real numbers $y \in \Re$	Range: $\{0, 4, 10, 14\}$

	
Domain: All real numbers greater than or equal to –3 and less than 8 $-3 \le x \le 8, x \in \Re$	Domain: All real numbers greater than –8 and less than or equal to 8 $-8 < x \le 8, x \in \Re$
Range: All real numbers greater than or equal to –5 and less than or equal to 6 $-5 \le y \le 6, y \in \Re$	Range: All real numbers greater than –6 and less than or equal to 2 $-6 < y \le 2, y \in \Re$

<table>
<tr><th>Continuous function</th><th>Discrete function</th></tr>
<tr><td>Perimeter in inches of a square with side length of x</td><td>Perimeter in inches of a square created by 1 inch square tiles</td></tr>
<tr><td>$f(x) = 4x$</td><td>$f(x) = 4x$, by increments of 1 inch square tiles</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Domain: All real numbers greater than 0
$x > 0,\ x \in \Re$</td><td>Domain: All integers greater than 0
$x > 0,\ x \in Z$</td></tr>
<tr><td>Range: All real numbers greater than 0
$y > 0,\ y \in \Re$</td><td>Range: All positive integers that are multiples of 4
OR
$y > 0,\ y \in Z$ and y is a multiple of 4
OR
$y > 0$, where $y = 4x$</td></tr>
</table>

1) A function is represented by the set of ordered pairs shown below.

{(-3, -4), (-1, 2), (4, 17), (8, 29), (14, 47)}

What is the domain of this function? What is the range of this function?

The domain is the set of all x-values in the function. Remember that a point is written in the form of (x, y). Therefore, the x value in the point (-3, 4) is -3 and the y value is 4. The x value in the point (-1, 2) is -1 and the y value is 2 and etc. So to find the domain of the above function we just need to find all of the x-values listed. The range is the set of all y-values in the function. So to find the range of the above function we just need to find all of the y-values listed.

It is common practice to write the domain and range from least to greatest order.

Domain: {-3, -1, 4, 8, 14}

Range: {-4, 2, 17, 29, 47}

2) The domain of the function y = -5x + 6 is {-17, -6, 3, 12}. What is the range of this function?

In the above problem we are given the domain as {-17, -6, 3, 12}. The domain is our input values, or in other words, are x-values. We are looking for our range, our output values(aka y-values). In order to find our range values we just need to simply plug in each of our domain values into the function and solve to find our range values.

Domain value	**Function** **y = -5x + 6**	**Range value**
-17	y = -5(-17) + 6	91
-6	y = -5(-6) + 6	36
3	y = -5(3) + 6	-9
12	y = -5(12) + 6	-54

Range: {-54, -9, 36, 91}

3) What is the domain of the function y = 3x + 6 when the range is {-6, -1, 3, 7, 10}

This problem is different from #2 as this time we are looking for the domain given the range. There is two different methods I like to use to solve this problem.

1st method: We could plug in our range members in for y and then solve for the x-value.

Function: y = 3x + 6	Range(-6)	Range(-1)	Range(3)	Range(7)	Range(10)
Step 1: Plug in	-6 = 3x + 6	-1 = 3x + 6	3 = 3x + 6	7 = 3x + 6	10 = 3x + 6
Step 2: Solve for x: Add or subtract Constants	-6 = 3x + 6 -6 - 6 -12 = 3x	-1 = 3x + 6 -6 - 6 -7 = 3x	3 = 3x + 6 -6 - 6 -3 = 3x	7 = 3x + 6 -6 - 6 -1 = 3x	10 = 3x + 6 -6 - 6 4 = 3x
Solve for x: Divide	$\frac{-12}{3} = \frac{3x}{3}$ -4 = x	$\frac{-7}{3} = \frac{3x}{3}$ $-\frac{7}{3}$ = x	$\frac{-3}{3} = \frac{3x}{3}$ -1 = x	$\frac{-1}{3} = \frac{3x}{3}$ $-\frac{1}{3}$ = x	$\frac{4}{3} = \frac{3x}{3}$ $\frac{4}{3}$ = x
Solution(domain values)	x = -4	x = $-\frac{7}{3}$	x = -1	x = $-\frac{1}{3}$	x = $\frac{4}{3}$

Range: $\{-4, -\frac{7}{3}, -1, -\frac{1}{3}, \frac{4}{3}\}$

2nd Method: My preferred method. Take the function and solve for x before plugging in the range value to find the new function.

y = 3x + 6
-6   - 6
y – 6 = 3x ⇒ $\frac{y-6}{3} = \frac{3x}{3}$ ⇒ $\frac{y-6}{3} = x$ ⇒ $x = \frac{y-6}{3}$

Range value	New Function $x = \frac{y-6}{3}$	Domain value
-6	$x = \frac{-6-6}{3}$	-4
-1	$x = \frac{-1-6}{3}$	$-\frac{7}{3}$
3	$x = \frac{3-6}{3}$	-1
7	$x = \frac{7-6}{3}$	$-\frac{7}{3}$
10	$x = \frac{10-6}{3}$	$\frac{4}{3}$

Range: $\{-4, -\frac{7}{3}, -1, -\frac{1}{3}, \frac{4}{3}\}$

4) The student council sent its members on four field trips during the school year. The number of buses needed to transport the members on each trip is a function of the number of members who went on each trip. This function consists of only the ordered pairs (52, 3), (72, 4), (86, 5) and (105, 6). What is the domain for this situation?

In this problem the members are the input values while the buses are the output values. We are given the ordered pairs {(52, 3), (72, 4), (86, 5), (105, 6)} and are asked to find the domain. This problem is very similar to question #1 in this packet. So, to find our domain members we just need to find all the x-values listed.

Domain: {52, 72, 86, 105}

Look below for question # 5

5) Mr. Plummer's unit quiz had 20 multiple choice problems. He awarded 5 points for each correct answer with no partial credit. Identify the variables in the problem situation. What function model could be used to represent this situation? What are the domain and range of the function model? What are the domain and range of the problem situation? How do they compare? Does the problem situation represent a continuous or discrete function?

The questions correct would be our input value(x-value) while the total points scored would be our output values(y-values) as the amount of points scored depends on the questions correct. Therefore, our function would be y = 5x.

Function model? Our function model is our function. Function model: y = 5x

What is the domain of function model? The domain of our function will not be the same as the domain for the problem situation as we are just referring to the function itself. The domain and range of any linear equation that is not being restricted in the graph, by a real world scenario, word problem, and etc. will always be all real numbers. Therefore, the Domain: x ϵ $\mathfrak{R}$.

What is the range of the function model? Look to explanation above. Range: y ϵ $\mathfrak{R}$.

What is the domain of problem situation? Since our domain is restricted to the correct answer choices and assuming Mr. Plummer is not giving partial credit our domain is the number of answer choices we can get correct between 1-20.

Domain: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Or Domain: $0 \leq x \leq 20, x \in \mathbb{Z}$

What is the range of problem situation? Our range is restricted by the domain. So our Range can only be multiplies of 5 as we are only awarded five points for each correct answer choice.

Range: {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100}

Or Range: $0 \leq y \leq 100, where\ y = 5x$

How do they compare? The function model can be all real numbers between 0 – 20 for the domain and 0 – 100 for the range. While in the problem situation the domain must be an integer between 0 – 20 and must be an integer of a multiple of 5 between 0 – 100.

Problem situation discrete or continuous? The problem situation would be discrete as when we plot the graph we would plot the set of points {(0,0), (1, 5), (2, 10), (3, 15), and etc…}.

Therefore, our values would be separate and not connected. So, by Definition we would have a discrete function.