A.6A Domain and Range of a Quadratic Function

Definitions:

Quadratic function – a second degree polynomial function that can be described by $f(x) = ax^2 + bx + c$, where $a \neq 0$ and the graph of the function is always parabolic or U-shaped.

Domain – set of input values for the independent variable over which the function is defined. aka: all the "x-values".

Range – set of output values for the dependent variable over which the function is defined. aka: all of the "y-values".

Continuous function – function whose values are continuous or unbroken over the specified domain.

Discrete function – function whose values are distinct and separate and not connect; values are not continuous. Discrete functions are defined by their domain.

Inequality representations –

Verbal description	Inequality Notation
x is all real numbers less than five	$x < 5, x \in \Re$
x is all real numbers	$x \in \Re$
y is all real numbers greater than -3 and less than or equal to 6	$-3 < y \leq 6, y \in \Re$
y is all integers greater than or equal to 0	$y \geq 0, y \in \mathbb{Z}$

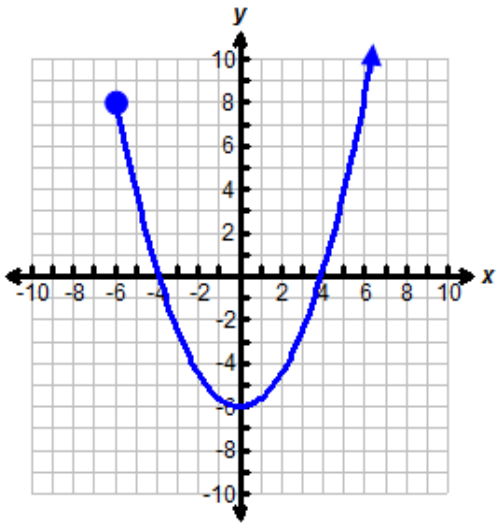
Note: Natural numbers are denoted by the symbol $\mathbb{N}$.

Whole numbers are denoted by the symbol W.

Integers are denoted by the symbol $\mathbb{Z}$.

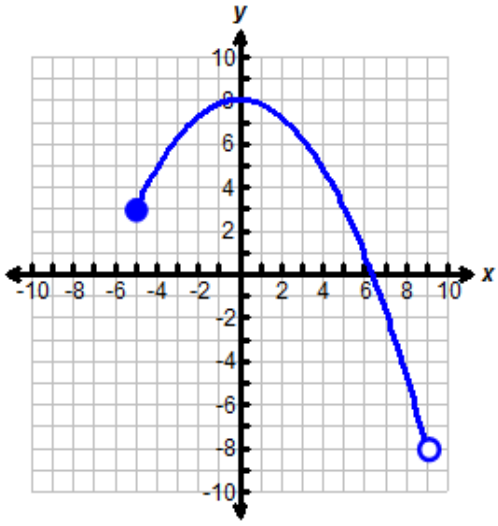
Real numbers are denoted by the symbol $\Re$.

Identify the domain and range for each graph below.



Domain of function:
All real numbers greater than or equal to –6
$x \geq -6$, $x \in \Re$

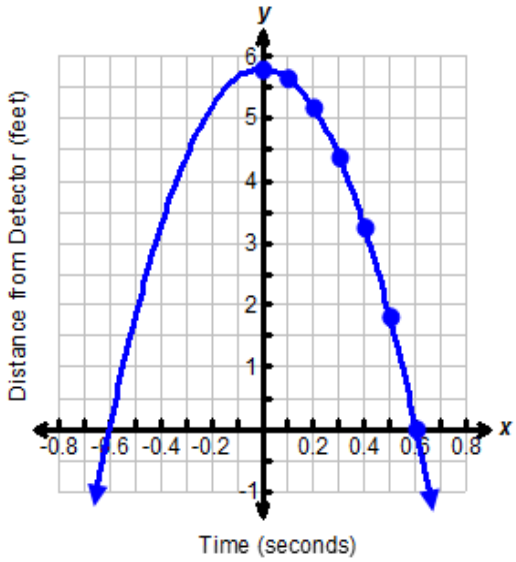
Range of function:
All real numbers greater than or equal to –6
$y \geq -6$, $y \in \Re$



Domain of function:
All real numbers greater than or equal to –5 and less than 9
$-5 \leq x < 9$, $x \in \Re$

Range of function:
All real numbers greater than –8 and less than or equal to 8
$-8 < y \leq 8$, $y \in \Re$

Harold used a motion detector to collect data on the distance a ball was from the ground after being dropped from a height of 5.8 feet as a function of time in seconds. The model function Harold calculated was $f(x) = -16x^2 + 5.8$. Create a graph of the model function. Determine the domain and range of the model function. Determine the domain and range of the problem situation. Explain why the domain and range of the model function and the domain and range of the problem situation are different.

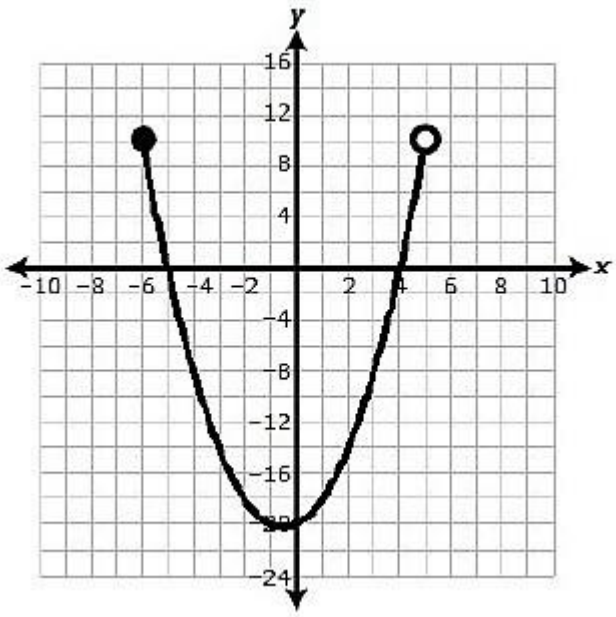


$f(x) = -16x^2 + 5.8$

Domain of model function: All real numbers $x \in \Re$	Domain of problem situation: All real numbers greater than or equal to 0 and less than or equal to 0.6 $0 \le x \le 0.6, x \in \Re$
Range of model function: All real numbers less than or equal to 5.8 $y \le 5.8, y \in \Re$	Range of problem situation: All real numbers greater than or equal to 0 and less than or equal to 5.8 $0 \le y \le 5.8, y \in \Re$

Sample response:
In the mathematical representation of the function, the parabola opens horizontally with no bounds; therefore, the domain is all real numbers. However, in the problem situation, time only goes from when the ball is dropped at 0 seconds until it hits the ground after an approximate elapsed time of 0.6 seconds; therefore, the domain is all real numbers greater than or equal to 0 seconds and less than or equal to 0.6 seconds.
Also, in the mathematical representation of the function the parabola has a maximum at 5.8 and continues down vertically with no bounds; therefore, the range is all real numbers less than or equal to 5.8. However, in the problem situation the ball is dropped from a height of 5.8 feet but cannot drop below ground level or 0 feet; therefore, the range is all real numbers greater than or equal to 0 feet and less than or equal to 5.8 feet.

1 A function is shown on the graph.



What is the domain of the function?

A The domain is all real numbers greater than –5 and less than 4.

B The domain is all real numbers greater than –6 and less than or equal to 5.

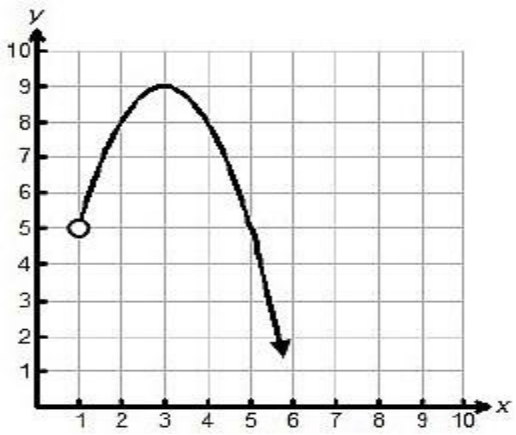
C $-6 \leq x < 5$

D $-20.25 \leq x \leq 10$

To determine the domain of the function we look at the graph from left to right and find all the possible x-values the graph contains. The furthest left point is (-6,10) where the graph contains the point as annotated with a solid circle. The furthest right point is (5, 10) where the graph does not contain the point as annotated with an open circle. The graph is a continuous graph as there is no break in the graph. So just looking at the x-values we are all real numbers between -6 to 5 but not including 5.

Therefore, the verbal description of the domain is all real numbers greater than or equal to -6 but less than 5. The inequality notation $-6 \leq x < 5$ would represent the domain of the function. C would be the correct answer choice.

2 A quadratic function is shown in the graph below.



What are the domain and range of the quadratic function?

A Domain: All real numbers less than or equal to 9

Range: All real numbers greater than 1

B Domain: All real numbers greater than 1
Range: All real numbers less than or equal to 9

C Domain: All real numbers greater than 1 and less than or equal to 6
Range: All real numbers greater than or equal to 0 and less than or equal to 9

D Domain: All real numbers greater than or equal to 1
Range: All real numbers less than 9

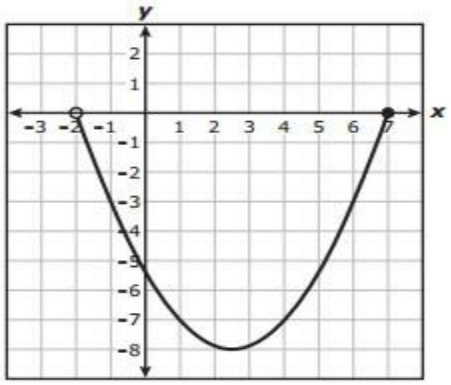
To determine the domain of the function we look at the graph from left to right and find all the possible x-values the graph contains. The furthest left point is (1, 5) where the graph does not contain the point as annotated with an open circle. The graph goes forever right and down as suggested by the graph having an arrow pointing down and right. The graph is a continuous graph as there is no break in the graph. So just looking at the x-values we are all real numbers between 1 but not including 1 to infinity(as we continue on forever right).

To determine the range of the function we look at the graph from up to down. We continually go down forever as suggested by the graph having an arrow point down and right. The furthest up point is (3,9) which is contained on the graph so we can be (3,9). So just looking at the y-values we are all real numbers between negative infinity(as we continue down forever) to 9.

Therefore, the verbal description of the domain is all real numbers greater than 1 and range is all real numbers less than or equal to 9. The inequality notation Domain: $x > 1$ and Range: $y \leq 9$ would represent the domain and range of the function. B would be the correct answer choice.

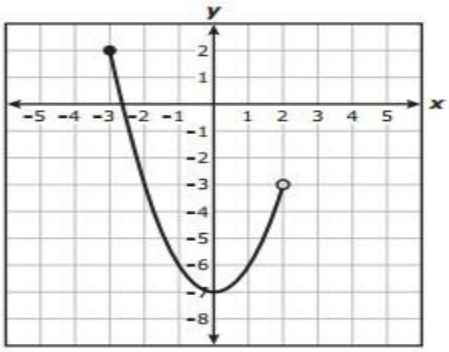
3 Which graph represents a function with a domain of all real numbers greater than or equal to -7 and less than 2?

A)



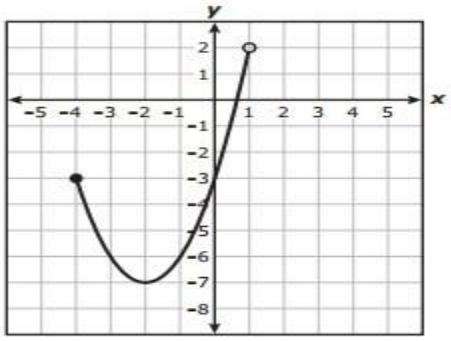
The domain of this graph looking from left to right would be all real numbers greater than -2 and less than or equal to 7. Therefore, this answer choice is not correct.

B)



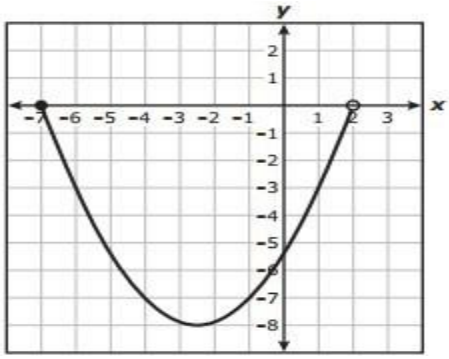
The domain of this graph looking from left to right would be all real numbers greater than or equal to -3 and less than 2. Therefore, this answer choice is not correct.

C)



The domain of this graph looking from left to right would be all real numbers greater than or equal to -4 and less than 1. Therefore, this answer choice is not correct.

D)



The domain of this graph looking from left to right would be all real numbers greater than or equal to -7 and less than 2. Therefore, this is the correct answer choice.

4 The graph below represents the height of a golf ball in feet as a function of the elapsed time since it was hit. The golf ball was in the air for 4 seconds.

What are the domain and range of the representative function?

A Domain: $0 \leq x \leq 80$
Range: $0 \leq y \leq 4$

B Domain: $0 < x < 80$
Range: $0 < y < 4$

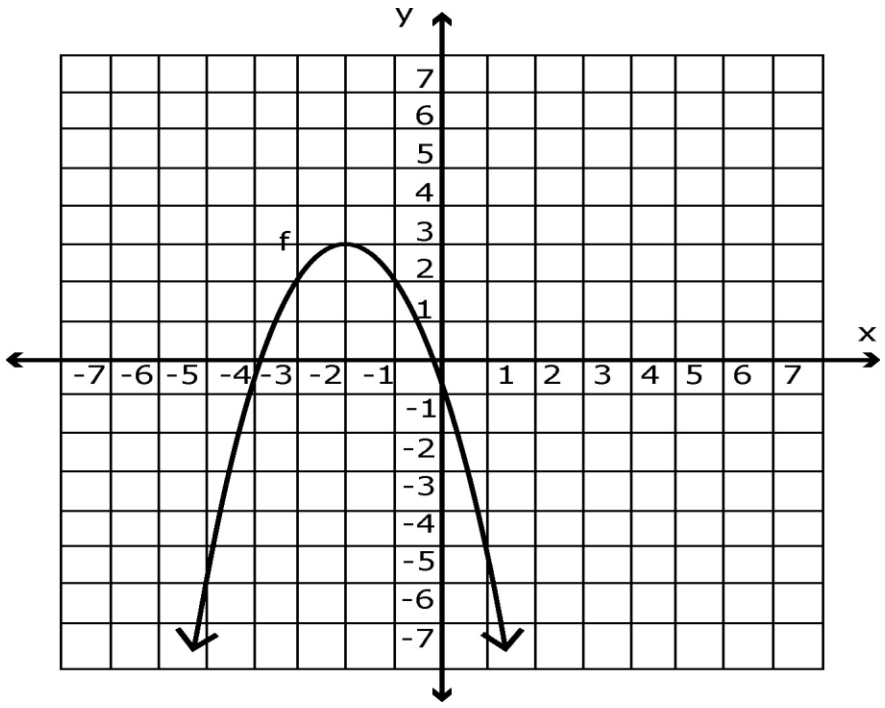
C Domain: $0 \leq x \leq 4$
Range: $0 \leq y \leq 80$

D Domain: $0 < x < 4$
Range: $0 < y < 80$

We can see that the function is continuous as there is no break in the graph. The ball is hit at 0 seconds and lands at 4 seconds. Our domain is represented by the elapsed time in seconds. Therefore, the domain is all real numbers greater than or equal to 0 and less than or equal to 4. The inequality notation $0 \leq x \leq 4$ would represent the domain of the function.

The height of the ball starts on the ground represented at 0 feet and is height to a max height of 80 feet in 2 seconds. Our range is represented by the height of the ball in feet. Therefore, the range is all real numbers greater than or equal to 0 and less than or equal to 80. The inequality notation $0 \leq x \leq 80$ would represent the range of the function. Therefore, C would be the correct answer.

5 What is the domain and range of the following quadratic function?



A Domain: $x \leq 3$
Range: $\infty < y < \infty$

B Domain: $-\infty < x < \infty$
Range: $y \leq 3$

C Domain: $y < 3$
Range: $\infty < x < \infty$

D Domain: $-\infty < x < \infty$
Range: $y < 3$

To find the domain we look from left to right. The function is not restricted and continues on forever in the left and right direction as annotated by the arrows going down and left and down and right in the graph. So, our domain would be x is all real numbers. The inequality notation $-\infty < x < \infty$ would represent the domain of the function.

To find the range we look from down to up. We see that our function goes down forever as annotated by the arrows in the graph. However, our graph has a maximum point of (-2, 3) and that point is contained in the function. So, our range would be y is all real number less than or equal to 3. The inequality notation $y \leq 3$ would represent the range of the function. Therefore, B would be the correct answer choice.