8.5F and 8.5H Distinguish Between Proportional and Non-Proportional Situation

Definitions:

Slope – the steepness of a line; rate of change in y(vertical) compared to change in x(horizontal, $or\ \frac{rise}{run}\ or\ \frac{change\ in\ y-values}{change\ in\ x-values}\ or\ \frac{(y_2 - y_1)}{(x_2 - x_1)}$, denoted as m in y = mx + b.

y-intercept – y coordinate of a point at which the relationship crosses the y-axis meaning the x coordinate is equal to zero, denoted as b in y = mx + b and the ordered pair (0, b).

Linear relationship – a relationship with a constant rate of change represented by a graph that forms a straight line.

- One quantity is dependent on the other
- Two quantities may be directly proportional to each other
- Can be classified as a positive or negative relationship
- Can be expressed as a pair of values that can be graphed as ordered pairs
- Graph of the ordered pairs matching the relationship will form a line

Function – relation in which each element of the input(x) is paired with exactly one element of the output(y).

- Linear proportional function
  - Linear
  - Passes through the point (0, 0)
  - Represented by y = kx
  - Constant of proportionality represented as $k = \frac{y}{x}$
  - When b = 0 in y = mx + b then k = the slope, m

- Linear non-proportional function

  - Linear
  - Does not pass through the origin (0, 0)
  - Represented by y = mx + b, where b $\neq$ 0
  - Constant slope represented as

$$m = \frac{rise}{run} \ or\ m = \frac{change\ in\ y\text{−}values}{change\ in\ x\text{−}values} \ or\ m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Linear Proportional Function	Linear Non-Proportional Function
Lindsey drove from her school to her aunt's house and recorded the amount of time she drove and the distance she travelled from the school. She began at her school and drove at a constant rate of 51 miles per hour. At 20 minutes, she was 17 miles from her school, at 30 minutes she was 25.5 miles from her school, and at one hour, she was 51 miles from her school.	Lindsey drove from her school to her aunt's house and recorded the amount of time she drove and the distance she travelled from the school. She began 10 miles from her school and drove at a constant rate of 51 miles per hour. At 20 minutes, she was 27 miles from her school, at 30 minutes she was 35.5 miles from her school, and at one hour, she was 61 miles from her school.

Linear Proportional Function	Linear Non-Proportional Function
Verbal Description: Lindsey started 0 miles from school. For each minute Lindsey drove, she travelled 0.85 miles.	Verbal Description: Lindsey started 10 miles from school. For each minute Lindsey drove, she travelled 0.85 miles.

Linear Proportional Function

Time in minutes (*x*)	Distance in miles (*y*)
0	0
20	17
30	25.5
60	51

Time in minutes (*x*)	Process	Distance in miles (*y*)
0	0(0.85)	0
20	20(0.85)	17
30	30(0.85)	25.5
60	60(0.85)	51

The data displayed in the table represents a linear proportional function because each independent value (*x*) is paired with exactly one dependent value (*y*), the ratio between the dependent and independent variables is a constant of proportionality (*k*), and the ordered pair (0, 0) exists.

$$(x,y) \rightarrow \frac{y}{x} = k$$

$$(20,17) \rightarrow \frac{17}{20} = 0.85$$

$$(30,25.5) \rightarrow \frac{25.5}{30} = 0.85$$

$$(60,51) \rightarrow \frac{51}{60} = 0.85$$

Linear Non-Proportional Function

Time in minutes (*x*)	Distance in miles (*y*)
0	10
20	27
30	35.5
60	61

Time in minutes (*x*)	Process	Distance in miles (*y*)
0	0(0.85) + 10	10
20	20(0.85) + 10	27
30	30(0.85) + 10	35.5
60	60(0.85) + 10	61

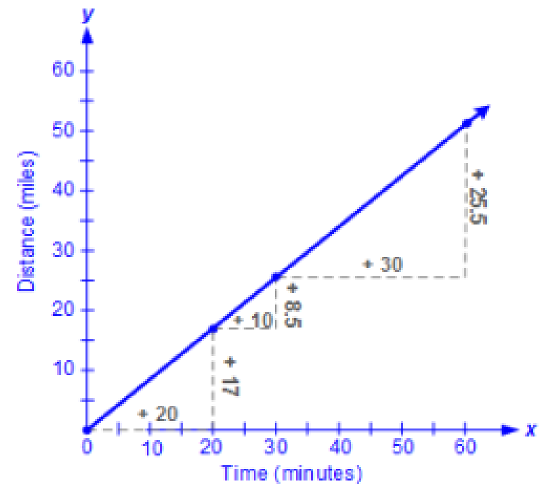
The data displayed in the table represents a linear non-proportional function because each independent value (*x*) is paired with exactly one dependent value (*y*), the ratio between the dependent and independent variables is not a constant of proportionality (*k*), and the ordered pair (0, 0) does not exist.

$$(x,y) \rightarrow \frac{y}{x} = k$$

$$(20,27) \rightarrow \frac{27}{20} = 1.35$$

$$(30,35.5) \rightarrow \frac{35.5}{30} \approx 1.18\overline{3}$$

$$(60,61) \rightarrow \frac{61}{60} \approx 1.01\overline{6}$$

<table>
<tr><th>Linear Proportional Function</th><th>Linear Non-Proportional Function</th></tr>
<tr><td>


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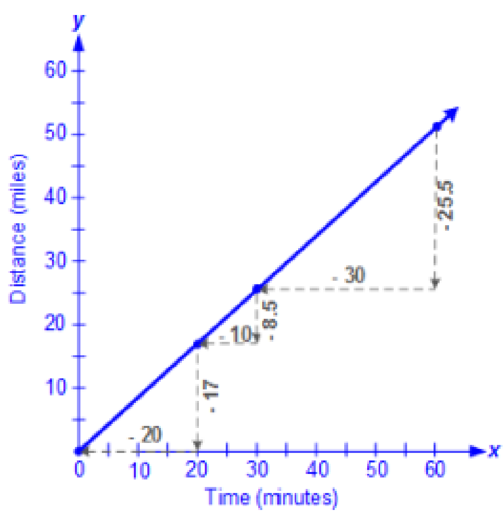

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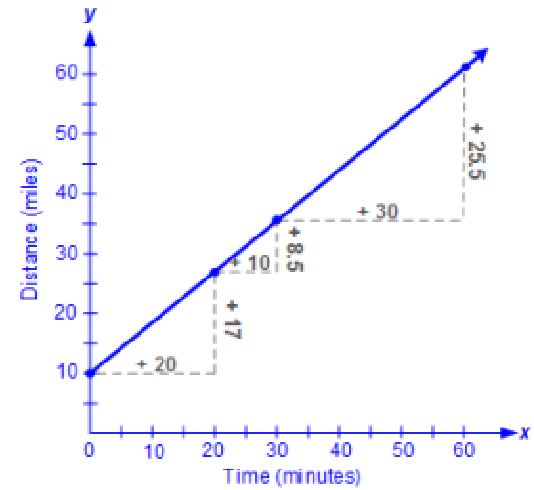

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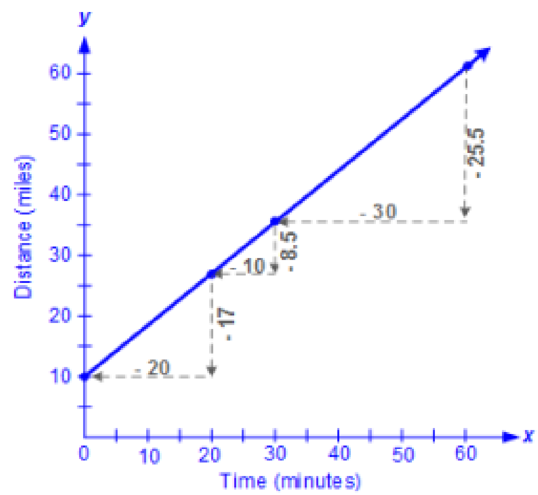

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The data displayed in the graph represents a linear proportional problem situation because the ratio between the dependent (*y*) and independent (*x*) variables is a constant of proportionality (*k*), the ordered pair (0, 0) exists, and the relationship forms a straight line when graphed.

$(x,y) \rightarrow \frac{y}{x} = k$

$(20,17) \rightarrow \frac{17}{-20} = 0.85$

$(30,25.5) \rightarrow \frac{25.5}{30} = 0.85$

$(60,51) \rightarrow \frac{51}{60} = 0.85$

OR

$(x,y) \rightarrow \frac{y}{x} = k$

$(-20,-17) \rightarrow \frac{-17}{20} = 0.85$

$(-30,-25.5) \rightarrow \frac{-25.5}{-30} = 0.85$

$(-60,-51) \rightarrow \frac{-51}{-60} = 0.85$
</td><td>
The data displayed in the graph represents a linear non-proportional problem situation because the ratio between the dependent (*y*) and independent (*x*) variables is not a constant of proportionality (*k*), the ordered pair (0, 0) does not exist, and the relationship forms a straight line when graphed.

$(x,y) \rightarrow \frac{y}{x} = k$

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OR

$(x,y) \rightarrow \frac{y}{x} = k$

$(-20,-27) \rightarrow \frac{-27}{-20} = 1.35$

$(-30,-35.5) \rightarrow \frac{-35.5}{-30} \approx 1.18\overline{3}$

$(-60,-61) \rightarrow \frac{-61}{-60} \approx 1.01\overline{6}$
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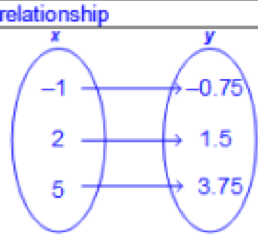
</table>

Linear Proportional Function	Linear Non-Proportional Function
$y = 0.85x$ where x represents time in hours, and y represents distance in miles	$y = 0.85x + 10$ where x represents time in hours, and y represents distance in miles
The equation represents a linear proportional function because it is written in the form $y = kx$.	The equation represents a linear non-proportional function because it is written in the form $y = mx + b$.

Linear proportional relationship

x	y
–1	–0.75
2	1.5
5	3.75



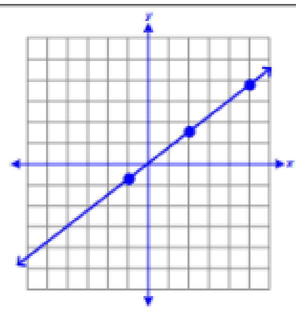
The data displayed in the table represents a linear proportional function because each independent value (x) is paired with exactly one dependent value (y), the ratio between the dependent and independent variables is a constant of proportionality (k), the ordered pair (0, 0) exists, and the relationship forms a straight line when graphed.

$(x, y) \rightarrow \frac{y}{x} = k$

$(-1, -0.75) \rightarrow \frac{-0.75}{-1} = 0.75$

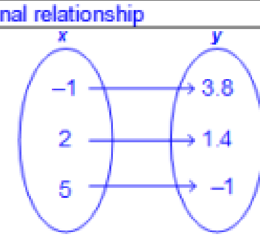
$(2, 1.5) \rightarrow \frac{1.5}{2} = 0.75$

$(5, 3.75) \rightarrow \frac{3.75}{5} = 0.75$



Linear non-proportional relationship

x	y
–1	3.8
2	1.4
5	–1



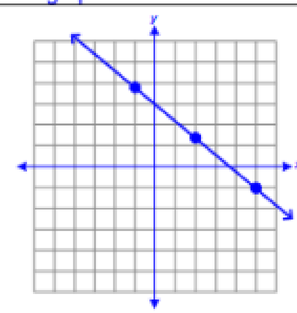
The data displayed in the table represents a linear non-proportional function because each independent value (x) is paired with exactly one dependent value (y), the ratio between the dependent and independent variables is a not a constant of proportionality (k), the ordered pair (0, 0) does not exist, and the relationship forms a straight line when graphed.

$(x, y) \rightarrow \frac{y}{x} = k$

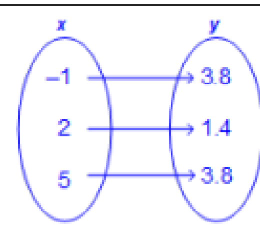
$(-1, 3.8) \rightarrow \frac{3.8}{-1} = -3.8$

$(2, 1.4) \rightarrow \frac{1.4}{2} = 0.7$

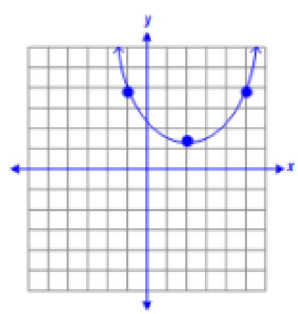
$(5, -1) \rightarrow \frac{-1}{5} = (-0.2)$



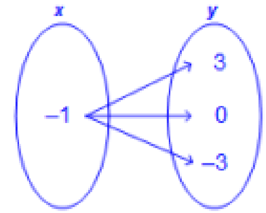
x	y
– 1	3.8
2	1.4
5	3.8



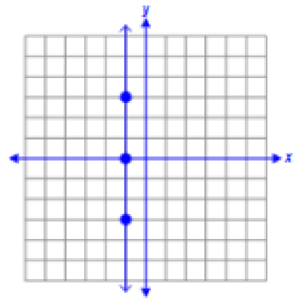
The table of data is a function, because each element of the input (x) is paired with exactly one element of the output (y); however, the table of data is neither a linear proportional nor a linear non-proportional function, because the data does not form a straight line when graphed.



x	y
–1	3
–1	0
–1	–3



The table of data is not a function, because the element of the input (*x*) is paired with different elements of the output (*y*); however, the table of data is a linear relationship, because the data forms a straight line when graphed.



Note: Please know that there are usually many different ways to solve an equation. If you watch the video lesson I will go through some of the different ways to solve the equations below including the use of technology.

1 Which equation <u>does not</u> represent a proportional relationship between x and y?

A $y = 0.75x$

B $y = 4x$

C $y = 1.5x$

D $y = 2x + 3$

To distinguish which of the above equations are proportional or non-proportional we need to remember that a proportional relationship is in the form of $y = mx + b$, where $b = 0$ or in the form $y = kx$. For a non-proportional relationship it is in the form of $y = mx + b$, where $b \neq 0$. In this case we are looking for the equation that is non-proportional as the problem asks for which equation does not represent a proportional relationship. The only equation that follows that form would be answer choice D as the b value is 3. In all the other equations given there is no constant value which indicates that the value of $b = 0$.

Answer choice D is the correct choice.

2 On a recent road trip, Judith recorded travel information as shown in the table below.

Judith's Travel Time

Time (minutes)	Distance (miles)
12	10
24	20
36	30
48	40

Determine if the relationship between time and miles traveled represent a proportional or a non-proportional relationship. Justify your response.

There are many different ways to solve this problem. Refer to the video lesson to get multiply ways to solve this problem.

To determine if the above equation is proportional or non-proportional let us find the equation for the given table in the form y = mx + b. Chose any two of the above points. For this problem I will let (24, 20) represent (x_2, y_2) and (12, 10) represent (x_1, y_1). Now let's find the slope using the following formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{20-10}{24-12} = \frac{10}{12} = \frac{5}{6}$$

Now plugging in our slope into the equation we have the following: $y = \frac{5}{6}x + b$

We not need to solve for the b value by choosing any of the points above and plugging them in for x and y and solving for b. It does not matter which point you chose as you the result will be the same regardless. For this problem I will chose the point (12, 10).

Step 1: Plug in (x, y) in our equation

$$y = \frac{5}{6}x + b$$

$$10 = \frac{5}{6}(12) + b$$

Step 2: Multiply

$$10 = 10 + b$$

Step 3: Solve for b

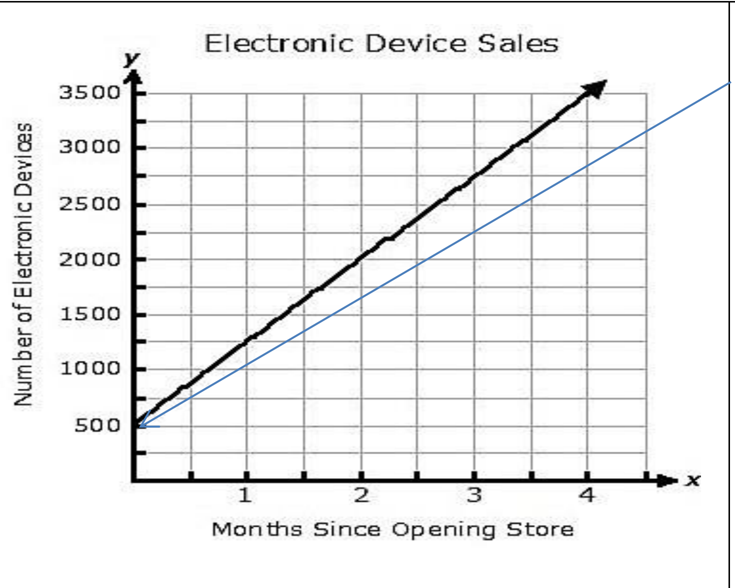
$$10 = 10 + b$$

$$\underline{-10 \quad -10}$$

$$0 = b$$

Since we have determined that b = 0, according to our definitions of proportional and non-proportional relationships we can determine that our table represents a **proportional relationship**.

3 After selling 500 electronic devices online, Theodore decided to open a store to sell electronic devices to in-store customers as well as online customers. The graph below shows Theodore's online and in-store electronic device sales according to the number of months since opening the store.



First thing we should notice is that the y-intercept is (0, 500). Since we do not pass through the origin(0,0) then by definition the graph of this function is a linear non-proportional relationship. Starting at the point (0, 500) and ending at the point (1, 1250) we can find the slope of the graph by using

$m = \frac{\Delta y}{\Delta x} = \frac{750}{1} = 750$

Therefore, by plugging in our slope and our y-intercept into the equation

y = mx + b we get the following:

y = 750x + 500

Which statement is true about the relationship of the data displayed in the graph?

A The data in the graph represents a linear proportional relationship because the constant of proportionality is 750 units per month and has a *y*-intercept of (0, 500).

B The data in the graph represents a linear proportional relationship because the data forms a straight line and does not include the ordered pair (0, 0).

C The data represents a linear non-proportional relationship because the data forms a straight line with a constant rate of change of 500 units per month and the graph does not include the ordered pair (0, 0).

D The data represents a linear non-proportional relationship because the data forms a straight line with the constant rate of change of 750 units per month and the graph does not include the ordered pair (0, 0).

We have determined that our graph represents a linear non-proportional relationship which leaves us with the options of answer choice C and answer choice D. Using what we found for the slope or our rate of change of 750, we can then determine that **answer choice D is the correct answer.**

4 Which relationship is an example of a proportional relationship?

A A rental car cost \$59 per day plus \$0.08 per mile driven.

B A gym membership has a \$45 registration fee and \$25 per month.

C The cell phone bill is \$30 plus \$0.10 for each call over 100 minutes.

D Samuel earns \$20 for each lawn he mows.

To determine which answer choice is the proportional relationship we can first translate each sentence into a mathematical sentence and then make a generalization.

A) Let the total cost of the rental car be represented by c and the total miles driven be represented my m, then $c = \$59 + \$0.08m$ or $c = \$0.08m + \$59$.
Since we have a y-intercept $\neq 0$, then the equation for answer choice A is a linear **non-proportional** relationship.

B) Let the total cost of gym membership be represented by y and the number of months be represented by x, the $y = \$45 + \$25x$ or $y = \$25x + \$45$.
Since we have a y-intercept $\neq 0$, then the equation for answer choice B is a linear **non-proportional** relationship.

C) Let the total cost of the cell phone bill be represented by b and number of minutes over 100 be represented by m, then $b = \$30 + \$0.10(100 - m)$ or $b = \$0.10(100 - m) + \$30$ such that $m > 100$.
Since we have a y-intercept $\neq 0$, then the equation for answer choice C is a linear **non-proportional** relationship.

D) Let the total amount Samuel earns be represented by e and the number of lawns he mows be represented by l, then $e = \$20l$
Since we have a y-intercept $= 0$, then the equation for answer choice D is a linear proportional relationship. **Answer choice D is the correct answer.**