A.3B Rate of Change of a Linear Function

Definitions:

Linear Function – a relationship with a constant rate of change represented by a graph that forms a straight line in which each element of the input(x) is paired with exactly one element of the output(y).

Rate of Change by Various Methods –

Tabular	Graphical	Algebraic
$m = \frac{\text{change in y}-\text{values}}{\text{change in x}-\text{values}}$ $m = \frac{\Delta y}{\Delta x}$ $m = \frac{y_2 - y_1}{x_2 - x_1}$	$\text{slope} = \frac{\text{rise}}{\text{run}}$	y = mx + b Solve equation for y. Slope is represented by m.

Sam's Big Eats has a sale of four dollars for five hot dogs. Complete the table and graph and label the points. Use the points to investigate slope by comparing rates of change in similar triangles.

Number of Hotdogs x	Cost in Dollars y
5	4
10	8

To investigate slope by comparing rates of change in similar triangles, label the following points on the graph above to compare changes in rise over run for points on the line.

Label:

- Origin as point *A*.
- (5, 4) as point *B*.
- (5, 0) as point *C*.
- (10, 8) as point *D*.
- (10, 0) as point *E*.

Compare rise over run:

$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{\text{rise}}{\text{run}} = \frac{\overline{DE}}{\overline{AE}} = \frac{8}{10} = \frac{4}{5}$$

$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{\text{rise}}{\text{run}} = \frac{\overline{BC}}{\overline{AC}} = \frac{4}{5}$$

Is $\triangle ABC$ similar to $\triangle ADE$? Explain your reasoning.

Yes, the triangles are similar.

Sample response: Angles are congruent. $\angle A \cong \angle A$; $\angle C \cong \angle E$ because both are right angles; and $\angle B \cong \angle D$ because the third angles are congruent if the other two are congruent; etc. The ratios of corresponding sides are equal, so sides are proportional: $\frac{4}{5}$ is equivalent to $\frac{8}{10}$; etc.)

Use similar triangles to explain why the slope is the same for any two points on the line representing the relationship between cost and number of hotdogs.

(Sample response: The ratio of the vertical rate of change to the horizontal rate of change is equal at any point along the line because each hotdog costs $\frac{4}{5}$ of a dollar or $0.80 and the cost is constant; etc.)

Determine the rate of change for the linear function represented by the following table of data.

x	*f*(*x*)
–4	–7
0	5
4	17

Algebraic:

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{17-5}{4-0}$$

$$m = \frac{12}{4}$$

$$m = 3$$

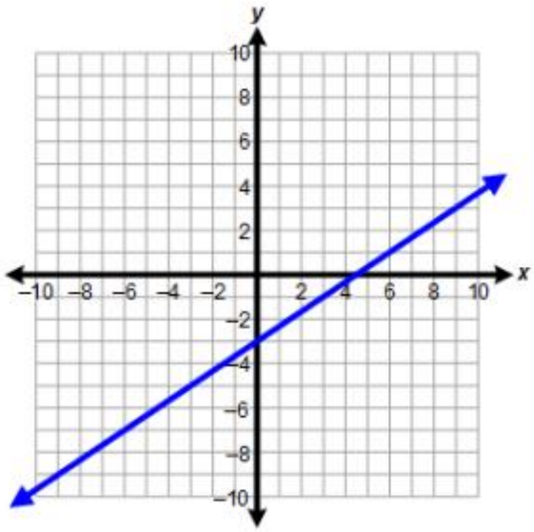
Graphing calculator:
Enter points the calculator and calculate the line of regression.

x	*y*
–4	–7
0	5
4	17

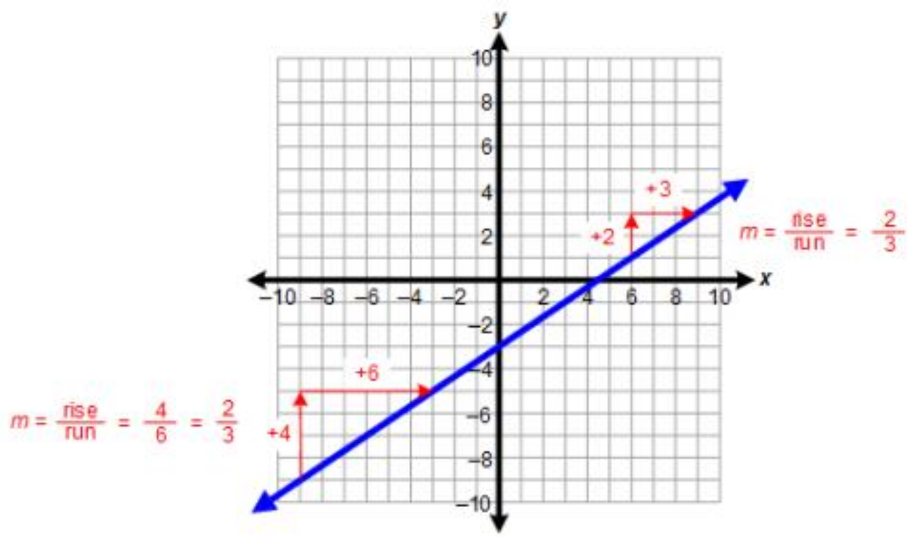
Linear Regression ($ax + b$)
regEQ(x) = $3x + 5$
$a = 3$
$b = 5$

Therefore, $m = 3$.

Determine the rate of change for the linear function represented in the graph below.



Sample response:



$\text{slope} = \frac{\text{rise}}{\text{run}}$

$m = \frac{2}{3}$

Determine the rate of change for the linear equation, $4x - 3y = 6$.

$4x - 3y = 6$

$-3y = -4x + 6$

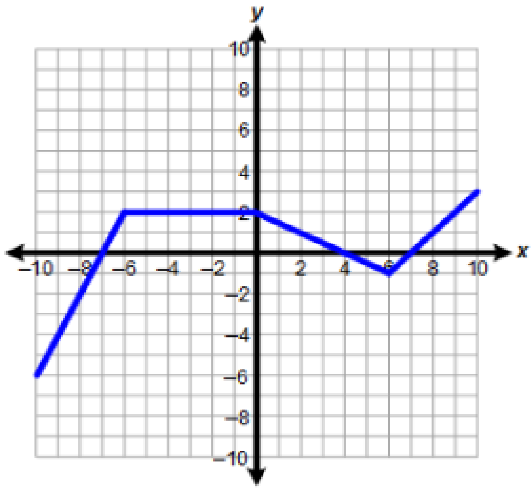
$y = \frac{4}{3}x - 2$

Since $y = mx + b$, the rate of change, $m = \frac{4}{3}$.

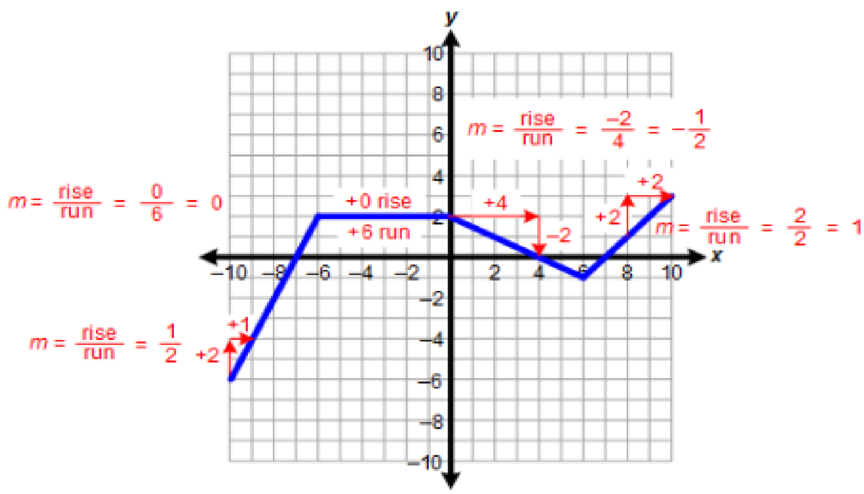
Determine the rate of change for the linear function, $f(x) = -3.5x + 7.25$.

Since $f(x) = mx + b$, the rate of change, $m = -3.5$.

Given the following graph



Calculate the rate of change over each interval.
Sample response:



Interval	Slope
$-10 \le x < -6$	2
$-6 \le x < 0$	0
$0 \le x < 6$	$-\frac{1}{2}$
$6 \le x < 10$	1

Compare the rates of change over the intervals of the graph.

Sample response:
The rate of change over the interval, $-10 \le x < -6$, has a slope of +2, indicating an increasing interval. The rate of change over the second interval, $-6 \le x < 0$, has a slope of 0, indicating that the y value remains constant throughout the interval.

The rate of change over the third interval, $0 \le x < 6$, has a slope of $-\frac{1}{2}$, indicating a decreasing interval.

The rate of change over the final interval, $6 \le x < 10$, has a slope of +1, indicating an increasing interval.

OR

The graph shows two increasing intervals, $-10 \le x < -6$ and $6 \le x < 10$, one constant interval, $-6 \le x < 0$, and one decreasing interval, $0 \le x < 6$.

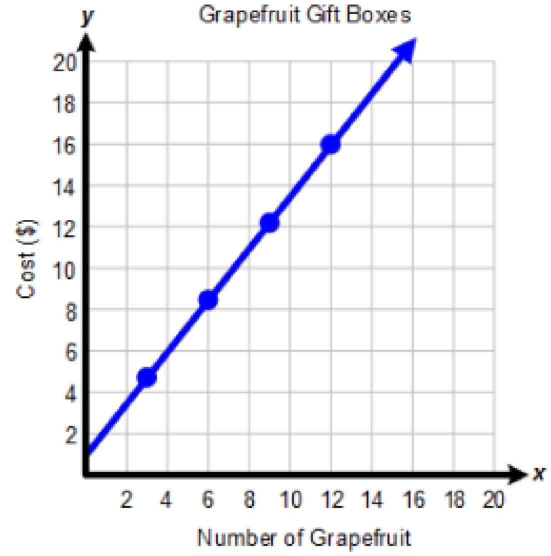
Describe a real-world situation that would represent the given graph.

Sample response:
A flying fish is 10 feet below the water. The fish surges upward at a constant rate for 4 seconds until it reaches a height of 2 feet above the water. The fish sails 2 feet above the water for 6 seconds. The fish then dives back into the water at a constant rate for 6 seconds, reaching a depth of 1 foot below the water. The fish then surges upward at a constant rate for 4 seconds until it reaches a height of 3 feet above the water.

Thriftway Grocery had the following sign posted for prices of grapefruit gift boxes.

Number of Grapefruit	Cost ($)
3	4.75
6	8.50
9	12.25
12	16

Create a graph to represent cost as a function of number of grapefruit.



Determine the rate of change of the linear function by two methods.

Sample methods:

$m = \frac{\text{rise}}{\text{run}}$

Using points (4, 6) and (12, 16), the rise is 5 units and the run is 4 units. This makes the slope $\frac{5}{4}$.

$(x_1, y_1) = (3, 4.75)$

$(x_2, y_2) = (6, 8.5)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{8.50 - 4.75}{6 - 3}$

$m = \frac{3.75}{3}$

$m = 1.25$

The slope of the function: $m = \frac{5}{4}$ or 1.25

Determine a function to represent cost as a function of number of grapefruit.

$f(x) = 1.25x + 1$

What does the rate of change represent in the problem situation?

The units on the rate of change are $\frac{\text{cost (\$)}}{\text{number of grapefruit}}$.

A slope of $\frac{5}{4}$ means four grapefruit cost $5 or $1.25 per grapefruit. The extra dollar represents the cost of packaging the gift box.

1 The amount of water that flows through the Hoover dam can be calculated using the following formula in which y equals the number of seconds and x equals the number of cubic feet: $6000x - 2y = 0$

What is the rate at which the water flows?

A 3,000 cubic feet per second

B 6,000 cubic feet per second

C 9,000 cubic feet per second

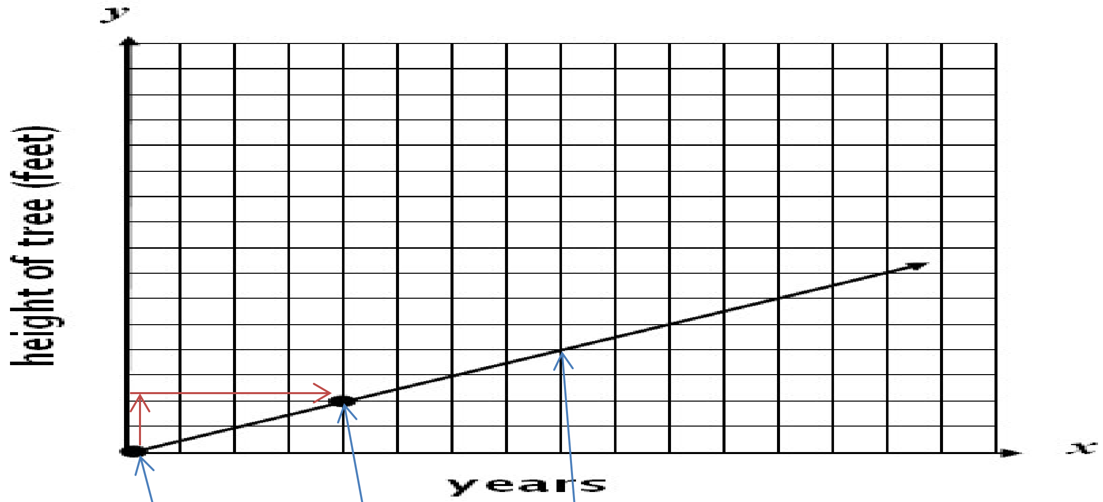
The simplest way to solve for rate of change would be to change our equation $6000x - 2y = 0$ written in standard form and change it to slope-intercept form ($y = mx + b$) as the m in the equation is our slope or also known as our rate of change. In order to change it to slope intercept form we must solve for the y-value.

Since we want to solve for y we need to get the y-value on one side of the equation. The numbers 6000x and -2 need to be taken to the other side of the equation.

Step:	Process:
Step 1: Subtract	$\begin{array}{rcl} 6000x - 2y &=& 0 \\ \underline{-6000x} & & \underline{-6000x} \\ -2y &=& -6000x \end{array}$
Step 2: Divide	$\frac{-2y}{-2} = \frac{-6000x}{-2}$
Step 3: Solve	$y = 3000x$

Now that our equation is written in slope-intercept form($y = mx + b$) we can see that our rate of change is 3000 feet per second as represented by the number in front of the x-value.

2 12 years ago Jonathan planted a maple tree. He plotted the height of the tree at the beginning of every summer and connected the growth with a line.



How fast is Jonathan's tree growing?

A 2 feet per year

B 1 inch every 2 years

C 6 inches per year

D 1 foot per year

We can find the rate of change by using two different methods. We can use the slope = $\frac{\text{rise}}{\text{run}}$ method as I have indicated using the red arrows or we can use the m = $\frac{y_2 - y_1}{x_2 - x_1}$ as explained below. Starting at the point (0,0) and going to the point (4,2) we can see that we go up 2 units(rise) and to the right 4 units(run). Going up and right is indicated with a positive number and down and left are represented with a negative number. Therefore using slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$ we see that the rate of change is $\frac{1}{2}$.

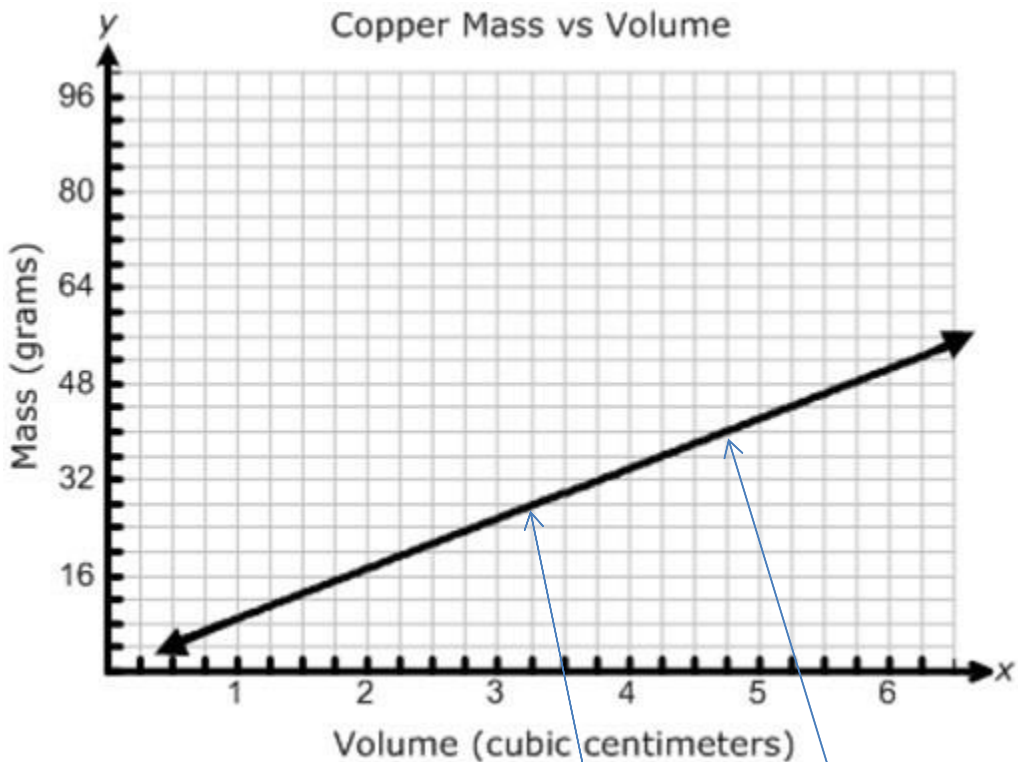
(0,0) (4,2) (8,4)

To find the rate of change let's first find two good points on the graph. When finding good points you want to preferable locate points where the graph crosses where the x and y grid lines intersect. It is not necessary but makes things easier as you will locate points involving integers and avoid fractions and/or decimals which may also be points where you are using best guess on the x and y values. You will only need two points and any two will do when trying to find the rate of change on a linear function as you will get the same result. In this case I will be using the points (0,0) and (4,2).

Now that I have two good points I will plug them into the equation m = $\frac{y_2 - y_1}{x_2 - x_1}$ in order to find my rate of change. The 2 and 1 in the equation are known as subscripts and they are only used to name the points. It does not matter which point you name your "1" point or "2" point. Once again you will get the same result. I will let my (x_2, y_2) be represented by the point (4,2) and (x_1, y_1) be represented by the point (0,0). Now plug your points into the formula and solve.

m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2} = \frac{\text{change in y-values}}{\text{change in x-values}}$; therefore, my rate of change is 1 inch for every 2 years.

3 The graph shows how the mass of copper changes as the volume of the element changes and the density remains constant.



Which of these best represents the rate of change in the mass of copper with respect to the volume?

(3.25, 28) (4.75, 40)

A $\frac{4}{33}$ g/cm^3 $\approx$ **.12**

B $\frac{19}{41}$ g/cm^3 $\approx$ **.46**

C $8\frac{1}{4}$ g/cm^3 **= 8.25**

D $4\frac{4}{7}$ g/cm^3 $\approx$ **4.57**

It is possible but more difficult to use the slope = $\frac{\text{rise}}{\text{run}}$ to find the rate of change for this graph.

Same as #2 above. Let's locate two good points and plug them into the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. The two points located above are not 100% accurate but are good estimations and we should be able to estimate our rate of change and make a good comparison to one of the answer choices. I will let my 2 point be represented by (4.75, 40) and my 1 point be represented by (3.25, 28).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40 - 28}{4.75 - 3.25} = \frac{12}{1.5} = 8$$

Results will vary depending on which two points you choose and how you estimate the points but as long as you make good estimations you should come close to one of the answer choices. In this case the answer choice C must closely matches what we have found.

4 Fancy Flower is a wholesaler that sells exotic flowers in 4-inch pots packaged in crates holding different numbers of pots. The table below represents the cost of four different sizes of crates that hold different numbers of 4-inch flower pots.

Number of 4-inch Flower Pots in the Crate	Cost of the Crate of 4-inch Flower Pots
9	$6.10
12	$7.30
16	$8.90
24	$12.10

Determine the rate of change of the cost of a crate of 4-inch flower pots with respect to the number of 4-inch flower pots in the crate using the table and graph.

Just as we did on questions #2 and #3 with the graphs we will just need to choose two points on the table to determine the rate of change. Once again it does not matter which two points you choose. To demonstrate that it does not matter I will do two different examples with different points.

Example 1: I will let my $(x_2,\ y_2)$ be represented by the point (9, \$6.10) and (x_1, y_1) be represented by the point (12, \$7.30).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.10 - 7.30}{9 - 12} = \frac{-1.2}{-3} = .4 = \frac{2}{5}$$

The rate of change indicates that for every inch increase the price will go up by \$.40 or looking at the fractional form for every 5 inches the price will go up by \$2.00.

Example 2: I will let my $(x_2,\ y_2)$ be represented by the point (24, \$12.10) and (x_1, y_1) be represented by the point (16, \$8.90).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12.10 - 8.90}{24 - 16} = \frac{3.2}{8} = .4 = \frac{2}{5}$$

Example 3: I will let my $(x_2,\ y_2)$ be represented by the point (24, \$12.10) and (x_1, y_1) be represented by the point (16, \$8.90).

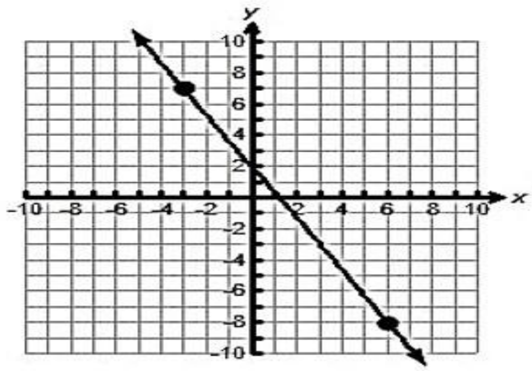
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12.10 - 8.90}{24 - 16} = \frac{3.2}{8} = .4 = \frac{2}{5}$$

In all examples we end up receiving the same result no matter which two points were used.

5 Which representations have the same rate of change of y with respect to x as the equation $3x + 5y = 15$?

I.	II.
$y = \frac{3}{5}x + 2$	$y + 7 = -0.6(x - 4)$

III.



IV.

x	y
−15	5
−10	2
−5	−1
0	−4
5	−7
10	−10

A II and IV only

B I, II, and IV only

C II, III, and IV only

D I, II, III, and IV

Let's first find the slope of our equation 3x + 5y = 15 written in standard form and rewrite the equation in slope-intercept form.

Step:	Process:
Step 1: Subtract	$3x + 5y = 15$ $\underline{-3x \qquad\quad -3x}$ $5y = 12x$
Step 2: Divide	$\frac{5y}{5} = \frac{-3x + 15}{5}$
Step 3: Solve	$y = -\frac{3}{5}x + 3$

As we can see the rate of change of the equation 3x + 5y = 15 is $-\frac{3}{5}$ as indicated by the number in front of the x variable.

Picture I: The slope or rate of change on I is $\frac{3}{5}$ as indicated by the number in front of the x variable in the equation y = $\frac{3}{5}$x + 2. Therefore, I <u>does not</u> have the same rate of change as our original equation.

Picture II: Picture II is written in slope intercept form($y - y_1 = m(x - x_1)$, where m represents the rate of change of the equation. In the equation $y + 7 = -0.6(x - 4)$ the rate of change is -0.6 which is the same as $-\frac{3}{5}$ when converted to a fraction. Therefore, II <u>does</u> have the same rate of change as our original equation.

Picture III: To find the rate of change for Picture III we can use slope = $\frac{\text{rise}}{\text{run}}$ or m = $\frac{y_2 - y_1}{x_2 - x_1}$. Using the two points given{(-3, 7) and (6, -8) on the graph we can see that we go down 15 units(rise) and go right 9 units. Therefore, slope = $\frac{\text{rise}}{\text{run}} = \frac{-15}{9} = -\frac{5}{3}$.

Or let (-3, 7) represent my 2 point and (6, -8) represent my 1 point then using

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-8)}{-3 - 6} = \frac{15}{-9} = -\frac{5}{3}$$

Therefore, III <u>does not</u> have the same rate of change as our original equation.

Picture IV: We can use m = $\frac{y_2 - y_1}{x_2 - x_1}$ to find the rate of change. Choose any two points. I will let (-15, 5) represent my 2 point and (-10, 2) represent my 1 point.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{-15 - (-10)} = \frac{3}{-5} = -\frac{3}{5}$$

Therefore, IV <u>does</u> have the same rate of change as our original equation.

Answer choice A would be the correct answer as Picture II and IV have the same representation of the rate of change as our original equation.